ETSAP Workshop and Training Course

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Primal and Dual Problems of Linear Programming Economic Models: Marginal Prices and Price Formation Equations

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Content

A – Why economic models for 4E analyses?

B – Why formulated as linear programs (LP)?

C – TIMES models: examples of shadow prices

D – List of common shadow prices (dual variables)

E – Price formation equations (dual constraints)

F – Practical tips

A1 – Why economic models for 4E analyses?

Why models? For carrying out mental experiments

Why mathematical? For assessing quantitatively the impact of choices

Why formal? For ensuring consistency, reproducibility and transparency

What choices?

- Socially well accepted
- Administratively implementable
- Economically affordable
- Technically feasible
- Environmentally sustainable

The economic consistency of possible development paths is not less important than the technical consistency

A2 – Why economic equilibrium models?

How to ensure that the present system, which by definition is in equilibrium, develops into a consistent system in 30, 50 or 100 year? The use of economic equilibrium models ensures that quantities and prices are feasible and in equilibrium in every year along the development path. [Simulation models cannot calculate prices.]

Economic models seem better than simulation models because:

- fully embody the technical aspects, while simulation models can embody the concept of cost, not price;
- ensure the economic feasibility in the long term, when the system is far away from the present conditions
- integrate the bottom-up and the top-down approach

A3 – Technology are ranked according to dynamic economic cost-benefit analyses

Economic equilibrium conditions determine what technologies are competitive, marginal or uncompetitive in each market. The evolution of investment costs, operation and maintenance costs, technical efficiencies and lifetime of each technology, together with the evolution of input and output energy commodity prices, determine changing cost-benefit conditions over time.

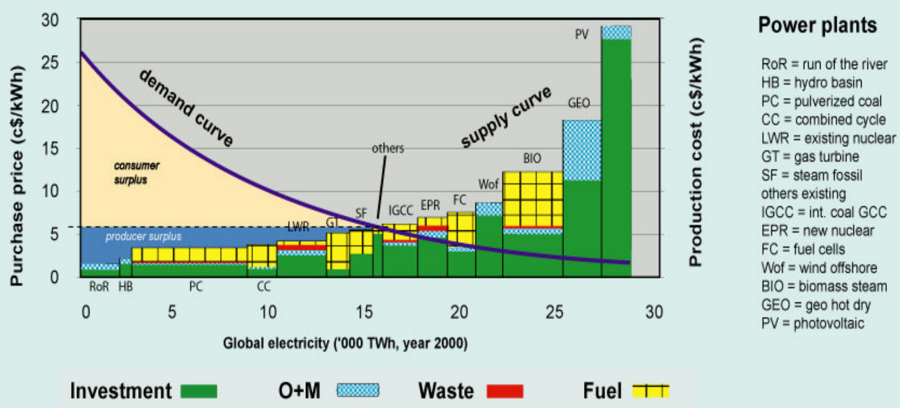
In a system view, where all energy commodities (flows and services) as well as all energy technologies are interlinked, the benefit-cost ratio provides a ranking indicator of energy supply technologies and end-use devices on a 'level playing field',. calculate simultaneously the equilibrium prices of all commodities in all years

B1 – Why formulated as linear programs?

- Simpler and clearer
- Detailed: supply and demand cost curves are specified as linear stepwise curves (see next slide)
- Technology explicit: each step in the curves corresponds to a name, a specific technology
- Huge dimensions and quick solutions
- Compact: the same information generates two separate problems
  1. On one side equations for physical quantities (primal problem)
  2. On the other side, equations for prices (dual problem)

It takes advantage of the special features of linear programs and their economic meaning (Von Neumann, 1931, 1936; Dantzig, 1949; Dorfmann, Samuelson, Solow, 1958; etc.)

B1a – (Inverse) supply curves formulated as technology explicit linear stepwise functions



Typical representation of an energy commodity in MARKAL - TIMES.
The algorithm maximises the global surplus over thousands such markets.

B2 – Compact matrix formulation of an LP problem

Composed of:	a Primal Problem and	a Dual Problem
	Max $c^t x$	Min $b^t y$
s.t.	$Ax \leq b$	$A^t y \geq c$
	$x \geq 0$	$y \geq 0$,

where:

- x is a vector of decision variables,
- $c^t x$ is a linear function representing the objective to maximize, and
- $Ax \leq b$ is a set of inequality constraints.

Each dual variable y_i may be assigned to its corresp. primal constraint.

Theor.1: If the primal problem has a finite, optimal solution x^*, then so does the dual problem (y^*); both problems have the same optimal obj. value.

Theor.2: When a resource i is not fully used, $A_i x < b_i$, its price y_i is zero; technologies j with a production cost greater than the price $A^t_j y > c_j$ do not produce ($x_j = 0$).

B3 – Detailed formulation

Duality in linear programming

Primal Problem

$$
\begin{array}{lllllllll}
\text{Min } c_1x_1 & + & c_2x_2 & + & & + & c_nx_n & & \\
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & \geq & b_1 \perp y_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & \geq & b_2 \perp y_2 \\
\vdots & & \vdots & & \ddots & & \vdots & & \vdots \quad \vdots \\
a_{i1}x_1 & + & a_{i2}x_2 & + & \cdots & + & a_{in}x_n & \geq & b_i \perp y_i \\
\vdots & & \vdots & & \ddots & & \vdots & & \quad \vdots \\
a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & \geq & b_m \perp y_m \\
 & & & & & & x_j & \geq & 0
\end{array}
$$

Dual Problem

$$
\begin{array}{lllllllll}
\text{Max } b_1y_1 & + & b_2y_2 & + & & + & b_my_m & & \\
a_{11}y_1 & + & a_{21}y_2 & + & \cdots & + & a_{m1}y_m & \leq & c_1 \\
a_{12}y_1 & + & a_{22}y_2 & + & \cdots & + & a_{m2}y_m & \leq & c_2 \\
\vdots & & \vdots & & \ddots & & \vdots & \leq & \vdots \\
a_{1j}y_1 & + & a_{2j}y_2 & + & \cdots & + & a_{mj}y_m & \leq & c_j \\
\vdots & & \vdots & & \ddots & & \vdots & & \\
a_{1n}y_1 & + & a_{2n}y_2 & + & \cdots & + & a_{mn}y_m & \leq & c_n \\
 & & & & & & y_j & \geq & 0
\end{array}
$$

Shadow price = change in objective function by increase of RHS by one unit

Decision variables = primal variables:
- Activity levels (usage of which technologies)
- Energy flows
- New capacity

Equations:
- Energy/emission balances
- Efficiency relationships
- Capacity-activity constraints
- Peak load constraint (ensuring reserve capacity)
- GHG mitigation target, Quota renewalbes,…

Decision variables = dual variables:
- Prices of energy carriers
- Opportunity costs for capacity
- Certificate prices for GHG, for renewable quotas,…

Equations:
- Comparison of prices and costs
- Assessment of cost-effectiveness of technology

B4 – Base assumptions of MARKAL-TIMES

Underlying principles central to the MARKAL-TIMES equilibria are:

- Outputs of a technology are linear functions of its inputs;
- Total economic surplus (or total utility) is maximised over the entire horizon (or total system cost is minimised)
- Energy markets are competitive, with perfect foresight (both cases with exceptions, see variants).

As a result of the assumptions the following properties hold:

- The market price of each commodity is exactly equal to its marginal value in the overall system, and
- Each economic agent maximizes its own profit (or utility).

B5 – Different levels of economic ‘rationale’

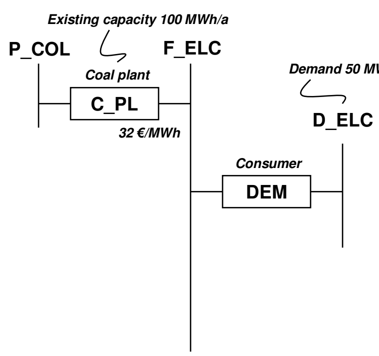
In principle there are 3 modes of building scenarios with MARKAL-TIMES models, depending on the scope of the system that participates to implementing policies:

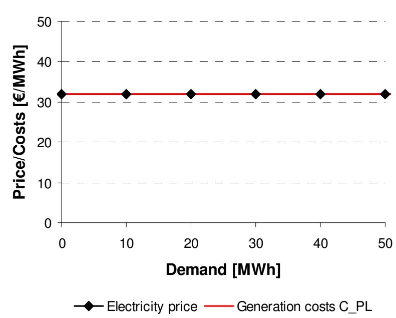
1. The least cost version (Standard MARKAL) evaluates the impact on the energy sectors, including the end-use devices;
2. The partial equilibrium version (MARKAL-ED; MARKAL-MICRO) includes in the evaluation the effects on the consumption levels;
3. The general equilibrium version (MARKAL-MACRO) evaluates the effects on the whole economy.

Any mode is compatible with any additional policy constraint (on emissions, energy dependence, efficiency, ...)

C1 – Examples of shadow prices (1)

Example: Electricity price (1)

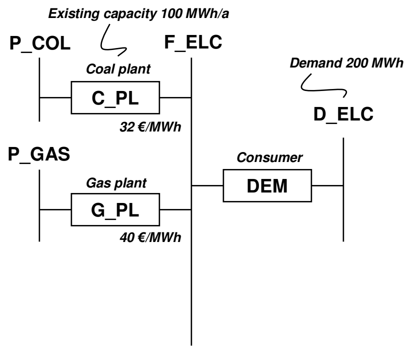


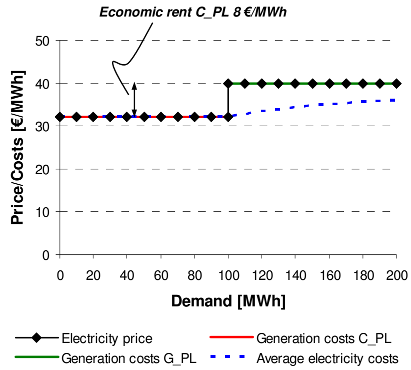


- Shadow price of electricity balance = Commodity price = cost increase by increasing production or reducing consumption by one unit
- Simple case: price equals average costs

C2 – Examples of shadow prices (2)

Example: Electricity price (2)





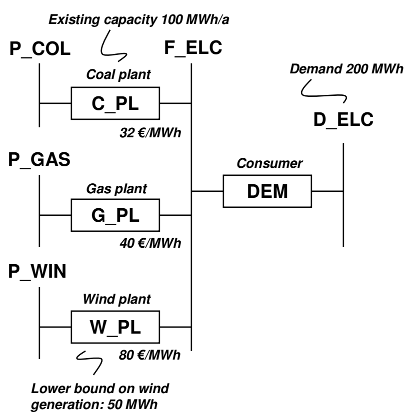
- Capacity of coal plant is limited to 100 MWh/a, no new capacity for coal plant allowed
- Gas plants sets the electricity price
- Capacity of coal plant earns an economic rent of 8 €/MWh = shadow price of capacity-activity constraint:

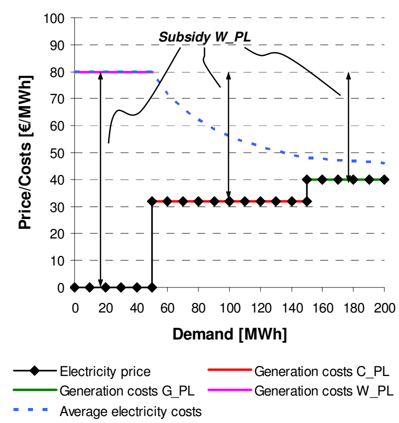
$$ACT_{C\_PL} \leq af_{C\_PL} pasti_{C\_PL} = 100\text{MWh}$$

- Price jump at a demand of 100 MWh

C3 – Examples of shadow prices (3)

Example: Electricity price (3)





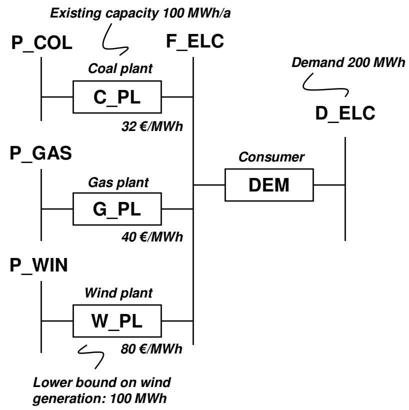
- Subsidy necessary to cover the higher costs of the wind plant
- Amount of subsidy = shadow price of lower bound on the activity variable = difference between costs of wind plant to electricity price:

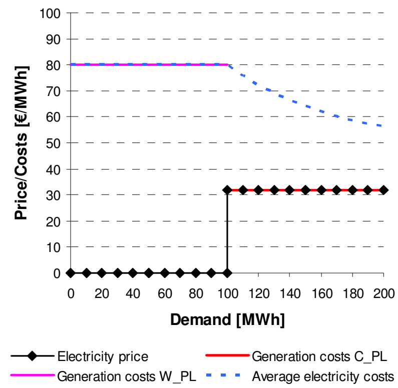
$$ACT_{W\_PL} \geq 50\text{MWh}$$

- Shadow price of lower bounds of zero on variables also called *reduced costs*

C4 – Examples of shadow prices (4)

Example: Electricity price (4)





- Large lower bounds on otherwise uneconomic options might lead to reducing electricity prices, since other plants are displaced (here: the gas plant)

C5 – Primal and dual problem of the example

Primal Problem

Min

$$\left(act\_cst_{C\_PL} + \frac{1}{\eta_{C\_PL}} price_{COL}\right) ACT_{C\_PL} + ncap\_cst_{C\_PL} NCAP_{C\_PL} + \left(act\_cst_{G\_PL} + \frac{1}{\eta_{G\_PL}} price_{GAS}\right) ACT_{G\_PL} + ncap\_cst_{C\_PL} NCAP_{C\_PL} \quad \text{Objective function}$$

$$-ACT_{C\_PL} + af_{C\_PL} NCAP_{C\_PL} \geq -af_{C\_PL} past_{C\_PL} \quad \perp \quad CAPACT_{C\_PL}$$

$$-ACT_{G\_PL} + af_{C\_PL} NCAP_{G\_PL} \geq 0 \quad \perp \quad CAPACT_{G\_PL}$$

$$ACT_{C\_PL} + ACT_{G\_PL} \geq dem_{ELC} \quad \perp \quad COMBAL_{ELC}$$

$$-NCAP_{C\_PL} \geq 0 \quad \perp \quad BNDNCAP_{C\_PL}$$

Dual Problem

Max

$$-af_{C\_PL} past_{C\_PL} CAPACT_{C\_PL} + dem_{elc} COMBAL_{ELC} \quad \text{Objective function}$$

$$-CAPACT_{C\_PL} + COMBAL_{ELC} \leq \left(act\_cst_{C\_PL} + \frac{1}{\eta_{C\_PL}} price_{COL}\right) \quad \perp \quad ACT_{C\_PL}$$

$$af_{C\_PL} CAPACT_{C\_PL} - BNDNCAP_{C\_PL} \leq ncap\_cst_{C\_PL} \quad \perp \quad NCAP_{C\_PL}$$

$$-CAPACT_{G\_PL} + COMBAL_{ELC} \leq \left(act\_cst_{G\_PL} + \frac{1}{\eta_{G\_PL}} price_{GAS}\right) \quad \perp \quad ACT_{G\_PL}$$

$$af_{G\_PL} CAPACT_{G\_PL} \leq ncap\_cst_{C\_PL} \quad \perp \quad NCAP_{G\_PL}$$

C6 – The aggregated TIMES matrix of the example

Variables

Equation sign

Equations

```
              |       C        O  O  O  O     |
              |       O        B  B  B  B     |
              |       M     N  J  J  J  J  O  |
              | A  C  N  F  C  I  F  V  S  B  |    R
              | C  A  E  L  A  N  I  A  A  J  |    H
              | T  P  T  O  P  V  X  R  L  z  |    S
----------------------------------------------------
EQ_OBJ        |                +  +  +  -  -  | E  0
EQ_OBJFIX     |             +     -           | E  -
EQ_OBJINV     |             +  -              | E  -
EQ_OBJSALV    |             +           -     | E  -
EQ_OBJVAR     | m        +           -        | E  0
EQ_ACTFLO     | +        -                    | E  0
EQL_ACTBND    | +                             | L  +
EQL_CAPACT    | +           -                 | L  +
EQE_CAPACT    | +           -                 | E  +
EQG_COMBAL    | m     +  m                    | G  +
EQE_COMBAL    | m     m  +                    | E  0
EQE_CPT       |    +        -                 | E  +
EQG_CPT       |             -                 | G  -
EQL_CPT       |             -                 | L  -
EQE_INSHR     |          m                    | E  0
EQ_IRE        | m                             | E  0
EQG_OUTSHR    |          m                    | G  0
EQE_OUTSHR    |          m                    | E  0
EQ_PTRANS     | +        m                    | E  0
EQG_UCRT      | m        +                    | G  0
EQL_UCR       | +                             | L  +
EQL_UCRT      | m        +                    | L  +
EQE_ACTEFF    | -        +                    | E  0
----------------------------------------------------
Variable Typ  | +  +  +  +  +  +  +  +  +  u
```


D1 – List of common shadow prices (dual variables)

Equation		Shadow price		
Name	**Description**	**Interpretation**	**Name in TIMES (lst, gdx file)**	**Name in VEDA-BE**
EQx_COMBAL	Commodity balance; EQE_COMBAL for strict equality, EQG_COMBAL for inequality	Commodity price; Costs of increasing production or reducing consumption by unit	EQx_COMBAL.M	EQ_COMBALM
EQ_PEAK	Peaking equation for a commodity; ensuring secure capacity of processes producing a certain commodity exceeds load by a given reserve margin in the timeslice with the highest load	*For consumers:* Premium on commodity price for consuming the commodity in the timeslice with the highest load; *For producers:* Contribution to cover investment and FOM costs	EQ_PEAK.M	EQ_PEAKM
EQ_PTRANS	Transformation equation, e.g. simple input-output relation: $FLO_{p,cout} - \eta_{p,cin,cout} \cdot FLO_{p,cin} = 0$	For simple relationship (example): cost change by increasing the efficiency $\eta_{p,cin,cout}$ by the amount $1/FLO_{p,cin}$	EQ_PTRANS.M	-
EQx_IN/OUTSHR	Upper/lower/fixed bound (x = L/G/E) on the share of an input/output flow in the total input/output of process, e.g.: $FLO_{p,c1} / (FLO_{p,c1} + FLO_{p,c2}) \leq flo\_shar_{p,c1}$	For upper bound share (example): part of the commodity price of *c1* is used to subsidy the production of commodity *c2* (subsidy for commodity *c2* = $flo\_shar_{p,c1} \cdot EQL\_INSHR.M$); for lower bound the subsidy flows the opposite direction	EQx_IN/OUTSHR.M	-
EQx_CAPACT	Capacity-activity constraint limits the activity of a process by its available capacity (x=L for upper bound, E for fixed availability factor)	Part of the commodity price which can be used to cover investment and FOM costs; value or economic rent of existing capacity obtained from commodity price	EQx_CAPACT.M	-
User constraints	Absolute bound type of user constraint	Cost change by increasing the RHS constant by one unit, e.g. marginal abatement costs of GHG limit	EQx_UC(R)T(S).M	User_conFXM, User_conLOM, User_conUPM
	Relative share type of user constraint, e.g. renewable electricity quota *q*: $\sum_{p \in ren} FLO_{p,elc} \Big/ \left(\sum_{p \in ren} FLO_{p,elc} + \sum_{p \in fos} FLO_{p,elc} \right) \geq q$	Certificate price of a quota system, where renewable generators receive the amount of *(1-q)x(shadow price)* and fossil producers have to pay *qx(shadow price)* per unit of electricity produced.	EQx_UC(R)T(S).M	User_conFXM, User_conLOM, User_conUPM
EQx_CUMPRD/CON	Cumulative bound over time on consumption or production of a commodity, e.g., fossil resource	Value of the limited resource, also called Hotelling rent, in addition to extraction costs	EQx_CUMPRD.M EQx_CUMCON.M	-
EQ_STGTSS	Storage process between timeslices with exogenous charging, e.g., hydro storage plant with natural inflow: $ACT_{p,s} - ACT_{p,s-1} + SIN_{p,s-1} - SOUT_{p,s-1} = stg\_chrg_{p,s-1}$	Value of the stored commodity, e.g. water	EQ_STGTSS.M	-
EQx_ACTBND or direct bound on activity variable	Bound on activity variable	Costs associated with the increase of the bound by one unit	VAR_ACT.M EQx_ACTBND.M	VAR_ACTM

D2 – List of common shadow prices (dual variables)

Equation		Shadow price		
Name	Description	Interpretation	Name in TIMES (lst, gdx file)	Name in VEDA-BE
EQx_CPT	Bound on total installed capacity	Costs associated with increase of the bound by one unit	EQx_CPT.M for upper and lower bounds VAR_CAP.M for fixed bounds	VAR_CAPM
Bound new capacity variable	Bound on new installed capacity	Costs associated with increase of bound by one unit	VAR_NCAP.M	VAR_NCAPM

E1 – Price formation equations (dual constraints)

Interpretation of dual equations

Dual Problem

$$\text{Max } b_1 y_1 + b_2 y_2 + \quad + b_m y_m$$

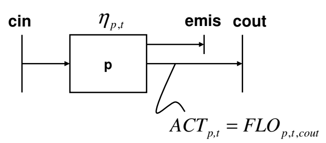
$$\begin{array}{ccccccccccc} a_{11}y_1 & + & a_{21}y_2 & + & \cdots & + & a_{m1}y_m & \leq & c_1 & \perp & x_1 \\ a_{12}y_1 & + & a_{22}y_2 & + & \cdots & + & a_{m2}y_m & \leq & c_2 & \perp & x_2 \\ \vdots & & \vdots & & \ddots & & \vdots & \leq & \vdots & \perp & \\ a_{1j}y_1 & + & a_{2j}y_2 & + & \cdots & + & a_{mj}y_m & \leq & c_j & \perp & x_j \\ \vdots & & \vdots & & \ddots & & \vdots & & & \perp & \\ a_{1n}y_1 & + & a_{2n}y_2 & + & \cdots & + & a_{mn}y_m & \leq & c_n & \perp & x_n \\ & & & & & & y_j & \geq & 0 & \perp & \end{array}$$

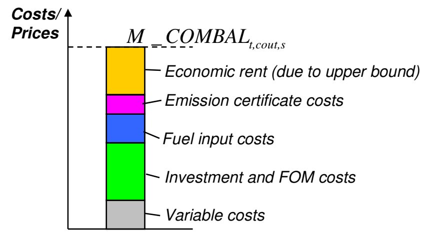
Complementarity theorem:

- *Each dual equations corresponds to one variable x of the primal problem.*
- *When the primal variable x is greater zero, dual equation is binding. Otherwise dual constraint is strict inequality*

- Two important dual equations:
  - Dual equation of activity (production) of technology describes: *if and how the product price is used to cover the technology costs (e.g. variable, capital costs, costs for CO_2 certificates etc.)*
  - Dual equation of investment variable describes: *how the capital costs of new installed capacity are recovered*

E2 – Dual equation: activity variable





Dual constraint:

Price of output commodity | *Variable operating costs* | *Contribution to investment and FOM costs* | *Fuel input costs* | *Subsidy* | *Economic rent (tax)*

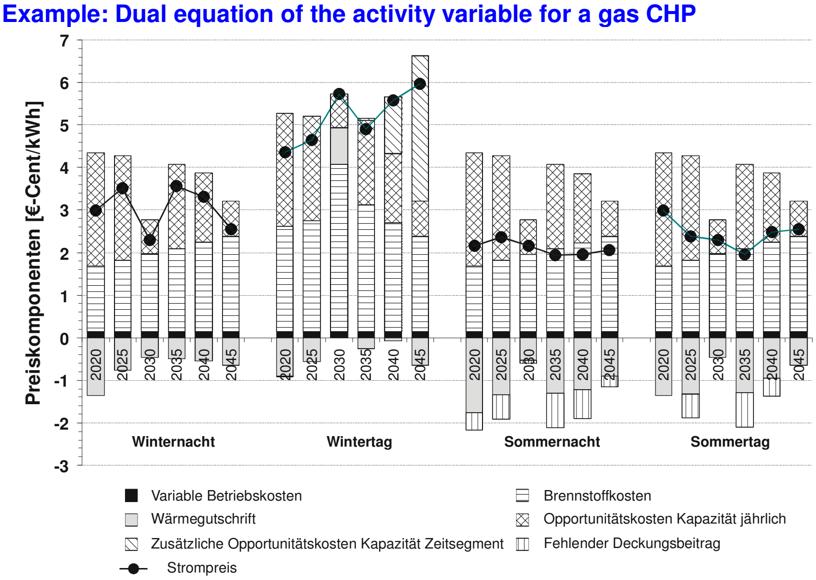
$$M\_COMBAL_{t,cout,s} = act\_cst_{t,p} + M\_CAPACT_{v,t,p,s} + \frac{1}{\eta_{t,p}} \cdot M\_COMBAL_{t,cin,s} - M\_BNDACT_{LO,t,p} + M\_BNDACT_{UP,t,p}$$

$$+ \frac{fac_{t,emis,p}}{\eta_{t,p}} \cdot M\_COMBNDPRD_{t,emis,s} + \begin{bmatrix} q_t \cdot M\_UC_{REN,t} \\ -(1-q_t) \cdot M\_UC_{REN,t} \end{bmatrix} + d_t \cdot M\_COMCUMPRD_{cout} + \ldots$$

Costs for emission certificates | *Upper term: Costs for renewable certificates. Lower term: Revenues from renewables certs.* | *Hoteling rent of cumulative resource constraint*

E3 – Example

Example: Dual equation of the activity variable for a gas CHP



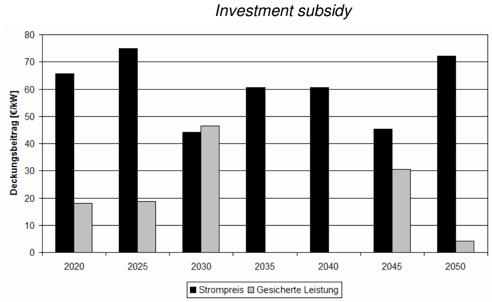
E4 – Dual equation: investment variable

Describes how the investment and FOM costs of new built capacity are covered

Discounted total investment costs — *Discounted FOM costs over lifetime* — *Contribution from production revenue* — *Contribution from providing secure capacity during peak demand*

$$inv\_cst_{p,v} + fom\_cst_{p,v} = \sum_{s\in prcts_p}\sum_{t\in vintyr_{p,t}} \alpha_{p,v,t,s}\cdot M\_CAPACT_{v,t,p,s} + \sum_{s\in comts_c}\sum_{v\in vintyr_{p,t}} \beta_{p,v,t,s}\cdot M\_PEAK_{v,t,p,s}$$
$$- M\_BNDNCAP_{LO,v,p} + M\_BNDNCAP_{UP,v,p} + \ldots$$

Investment subsidy — *Investment tax*



$$\alpha_{p,v,t,s} = cpt_{v,t,p}\cdot prc\_capact_p \cdot af_{v,p,s}\cdot \Delta s$$

$$\beta_{p,v,t,s} = \frac{cpt_{v,t,p}\cdot prc\_capact_p \cdot af_{v,p,s}\cdot \Delta s}{1 + com\_pkrsv_{t,c}}$$

E5 – What happens when a variable is bounded? The role of reduced costs

When the production cost of an activity variable is higher than the marginal price of the commodity, the variable reduces its production to the lowest possible level; the reduced cost quantifies the unit loss.

When the production cost of an activity variable is lower than the marginal price of the commodity, the variable increases its production to the highest possible level; the reduced cost quantifies the unit profit (or surplus).

In the output files, reduced costs are read in the column labeled “marginal” of the section “variables”

F1 – Practical tips: solve options

GAMS and CPLEX options

- Once can verify duality theorem and check convergence of model by adding OPTION SYSOUT=ON in the run file to get solution statistics

- (Intermediate model runs with option BARCROSSALG –1 in the cplex.opt file turns off crossover to basis solution and reduces computation time (for XPRESS solver crossover 0)).

```
Reading data...
Starting Cplex...
Tried aggregator 1 time.
LP Presolve eliminated 15481 rows and 13645 columns.
Aggregator did 6208 substitutions.
Reduced LP has 6068 rows, 13437 columns, and 60877 nonzeros.
.
.
Itn      Primal Obj        Dual Obj  Prim Inf Upper Inf  Dual Inf
   0  6.2007106e+010 -9.8309093e+012 2.31e+007 5.97e+006 6.31e+008
   1  5.6934408e+010 -8.9945079e+012 1.98e+007 5.12e+006 5.83e+008
   2  5.3911666e+010 -8.0489819e+012 1.73e+007 4.49e+006 5.30e+008
   3  5.5933399e+010 -6.7642026e+012 1.53e+007 3.97e+006 4.67e+008
   4  6.6348861e+010 -5.3673314e+012 1.39e+007 3.59e+006 4.20e+008
   5  7.3688350e+010 -3.9244872e+011 1.28e+007 3.31e+006 3.43e+008
  .
  .
  .
  68  3.1178168e+008  3.1178165e+008 1.24e-006 6.53e-009 4.13e-007
  69  3.1178167e+008  3.1178167e+008 1.03e-006 8.35e-008 1.10e-007
  70  3.1178167e+008  3.1178167e+008 8.19e-007 2.03e-007 4.31e-008
  71  3.1178167e+008  3.1178167e+008 8.97e-007 3.02e-007 3.09e-008
  72  3.1178167e+008  3.1178167e+008 1.21e-006 2.50e-007 2.69e-008
  73  3.1178167e+008  3.1178167e+008 1.00e-006 4.93e-008 5.76e-008

Optimal solution found.
Objective :   311781672.051982
```


F2 – Practical tips: where are shadow prices read?

- Shadow prices in the lst or gdx file are found under the marginal column:

```
---- EQU EQG_COMBAL  Commodity Balance (=G=)
                                  LOWER          LEVEL          UPPER        MARGINAL
IND.2000.AGR        .ANNUAL     537.2168       537.2168         +INF          23.9125
IND.2000.GHG        .ANNUAL         .              .            +INF      2.683327E-10
IND.2000.AGRBIO     .ANNUAL         .              .            +INF             EPS
IND.2000.AGRCOA     .ANNUAL         .              .            +INF             EPS
IND.2000.AGRDST     .ANNUAL         .              .            +INF          18.7631
IND.2000.AGRELC     .ID             .              .            +INF          24.5634
IND.2000.AGRELC     .IN             .              .            +INF          24.5634
IND.2000.AGRELC     .SD             .              .            +INF          24.5634
IND.2000.AGRELC     .SN             .              .            +INF          24.5634
IND.2000.AGRELC     .WD             .              .            +INF          24.5634
IND.2000.AGRELC     .WN             .              .            +INF          24.5634
```

- Shadow prices of equations or reduced costs of variables are discounted and valid for an entire period, while shadow prices in VEDA are undiscounted annual prices, e.g., in VEDA the commodity prices are already divided by the discount factor and the period duration

F3 – Practical tips: where are shadow prices read?

• Shadow prices in the lst or gdx file are found under the marginal column: