# Methods for analysing LP energy system models

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**ETSAP Workshop**

**25 November, Florence**

# Some approaches to analysis LP energy system models

**Balance model** → Calculation of commodity and technology specific average indicators under consideration of full process chain, e.g. primary energy consumption per demand of person kilometre

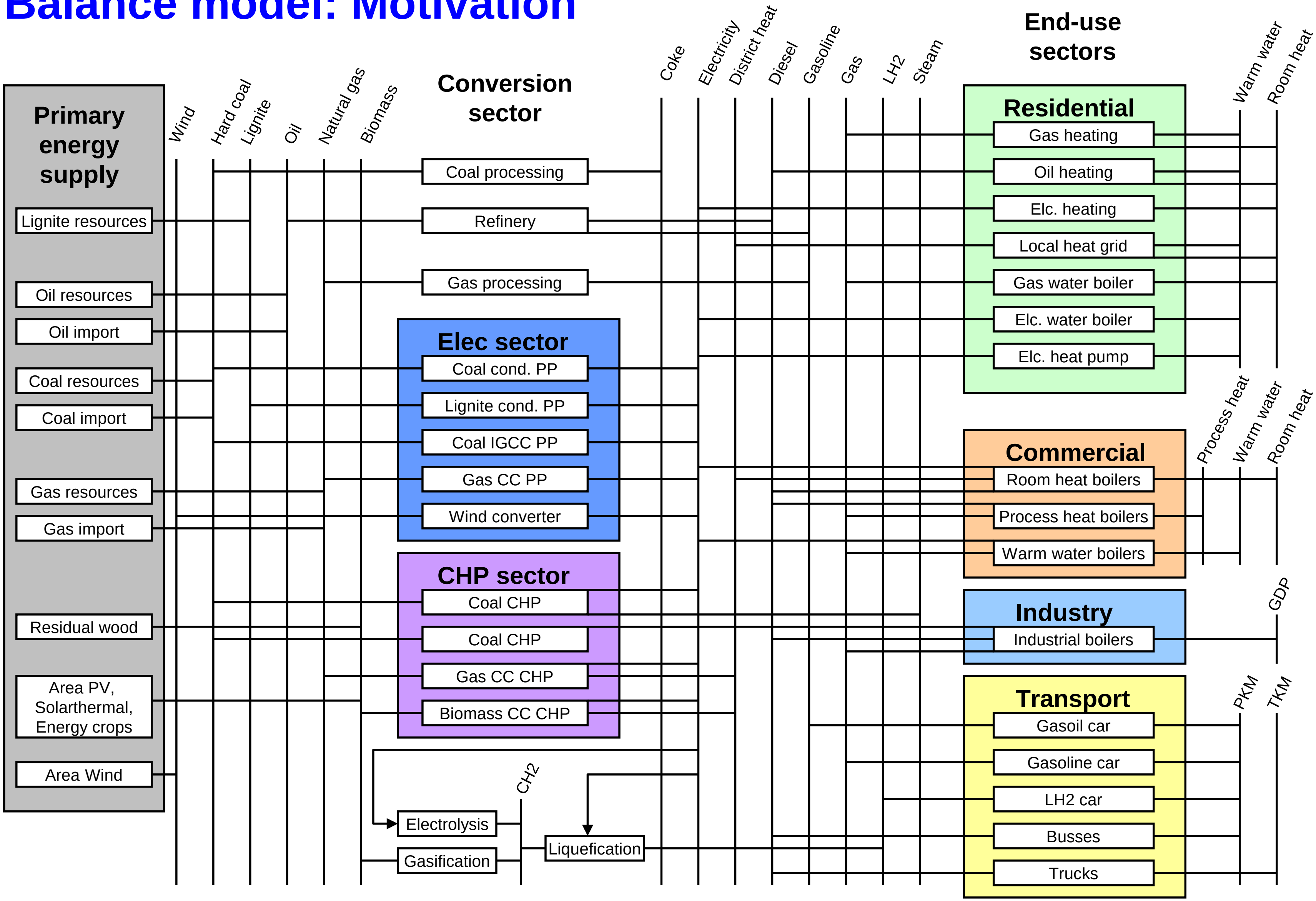
**Analysis of dual problem** →
- Evaluation of resources and constraints through prices, e. g. marginal abatement costs, value of a capacity unit
- Analysing the competitiveness of a technology taking into account costs and constraints as emission bounds or quotas

**Marginal sensitivity analysis** → Analysing the effect of marginal changes in the model input data $A,b,c$ on the solution

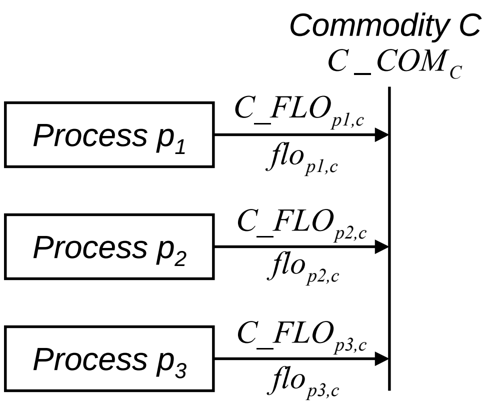
**Parametric Programming** → Variation of model input data $A,b,c$ over larger value ranges

$$\begin{aligned} &Min \quad c^T x \\ &\text{s.t.} \\ &Ax \geq b \\ &x \geq 0 \end{aligned}$$

# Balance model: Motivation



# Balance model: Equation system for costs

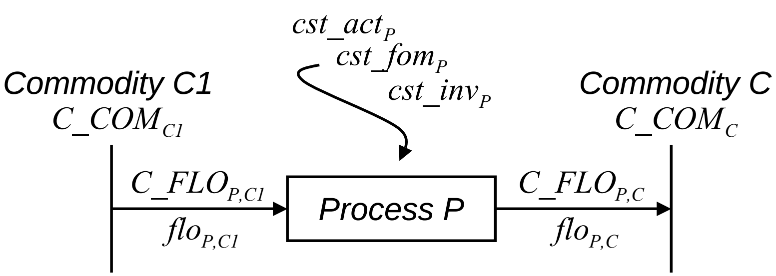



### *Balance equation for a commodity:*

$$C\_COM_C = \frac{\sum_{p \in top_{p,c,'out'}} flo_{P,C} \cdot C\_FLO_{P,C}}{\sum_{P \in top_{p,c,'out'}} C\_FLO_{P,C}}$$

Input parameters: *process flows* $flo_{p,c}$

Variables: *commodity generations costs* $C\_COM_c$




### *Balance equation for a process:*

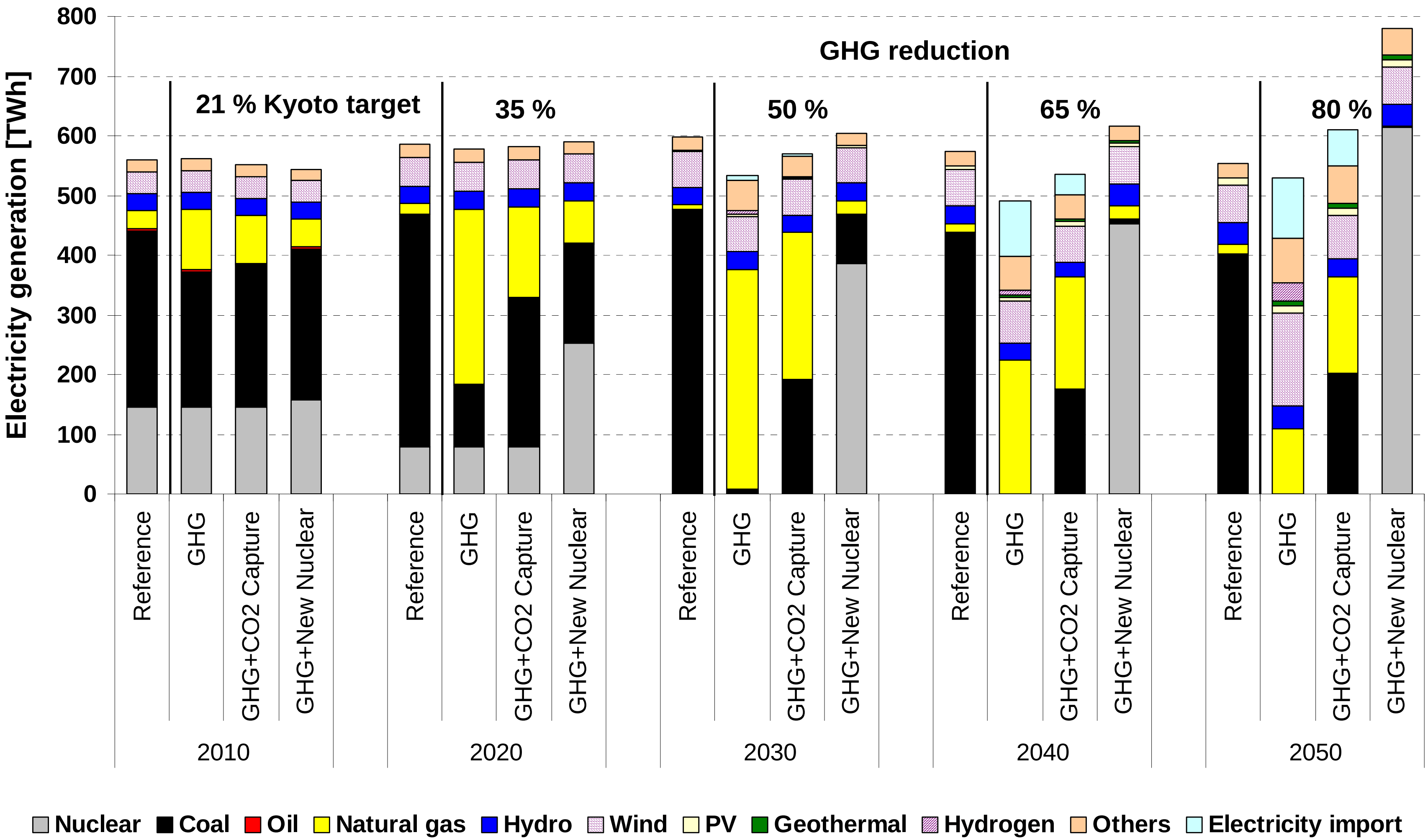
$$C\_FLO_{P,C} = \frac{\sum_{C1 \in top_{P,C1,'in'}} \left(flo_{P,C1} \cdot C\_COM_{C1}\right) + cst\_act_P + cst\_fom_P + cst\_inv_P + ire\_price_{P,C,R}}{flo_{P,C1}}$$

Input parameters: *process flows* $flo_{p,c}$*, annual activity costs* $cst\_act_{p,c}$*, annual investment cost* $cst\_inv_{p,c}$*, annual FOM costs* $cst\_fom_{p,c}$ *and imp/export prices* $ire\_price_{P,C,R}$ *for trade with region R*

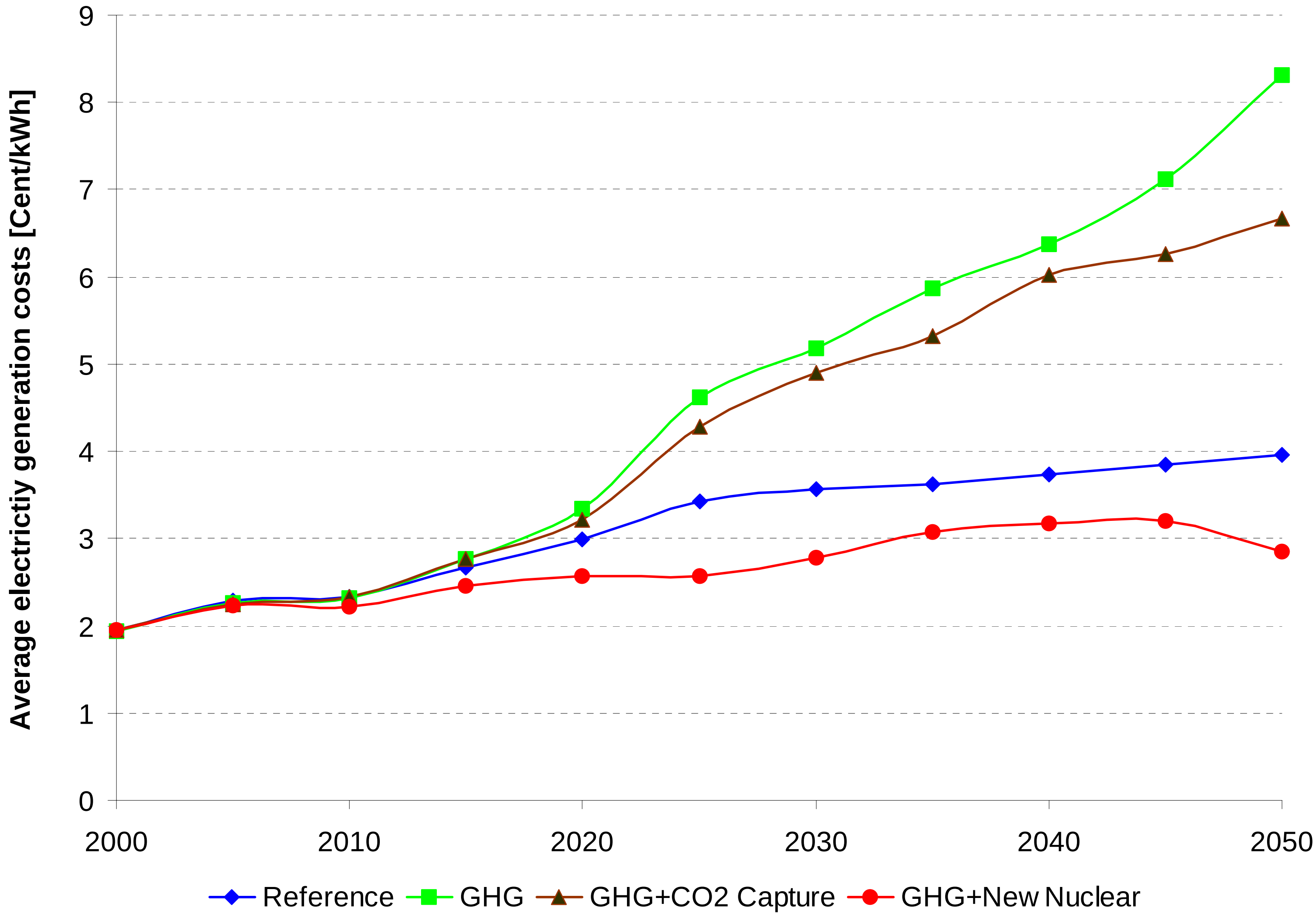
Variables: *commodity generations costs* $C\_COM_c$ *and the flow generation costs* $C\_FLO_{P,C}$

- **Square system of equations solved in GAMS with a dummy objective function**
- **Special treatment for processes with multiple outputs**
- **Similar system for emissions and primary energy consumption by fuel**

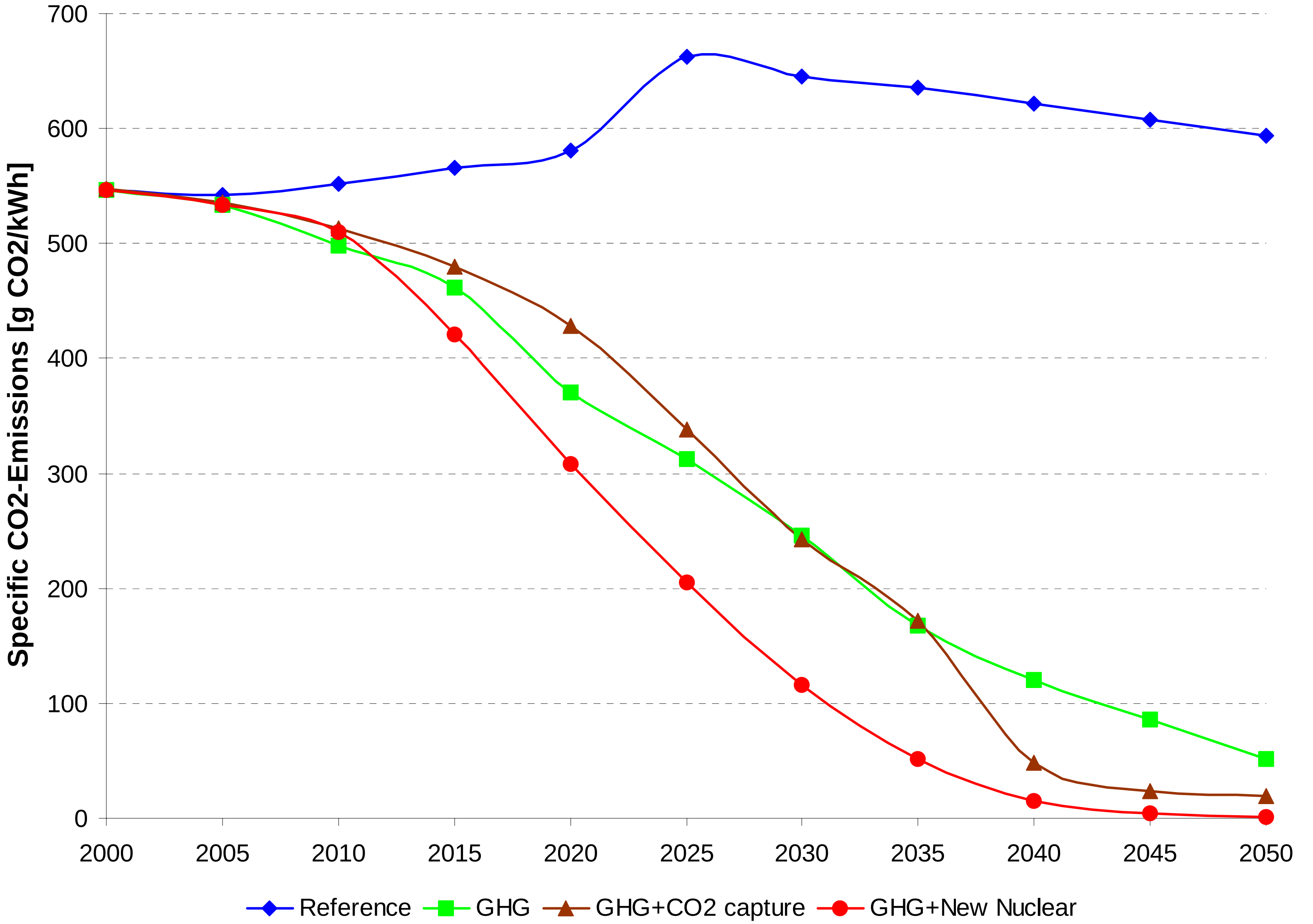
# Example: GHG reduction scenarios for Germany with TIMES



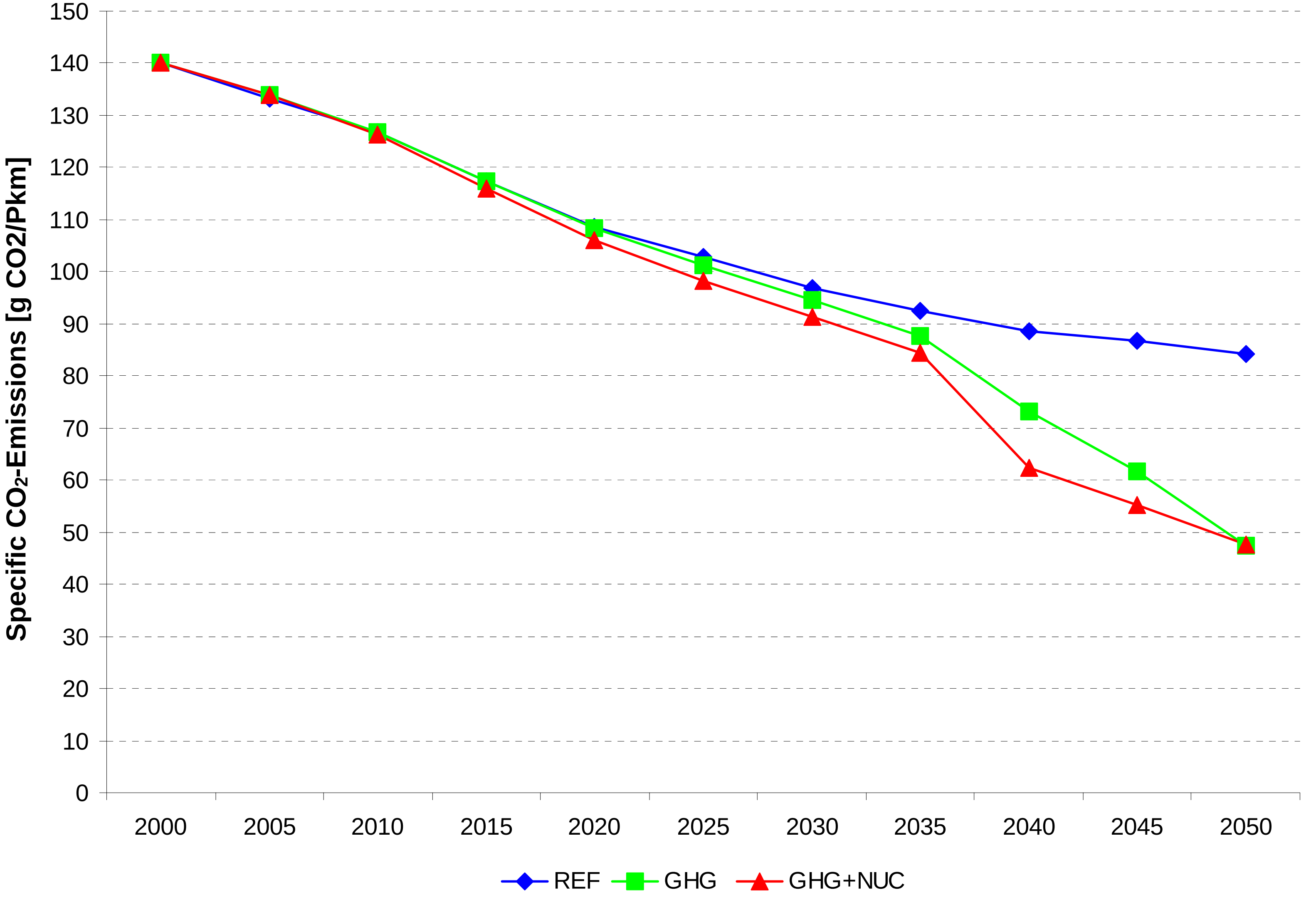
# Balance model: Electricity generation costs



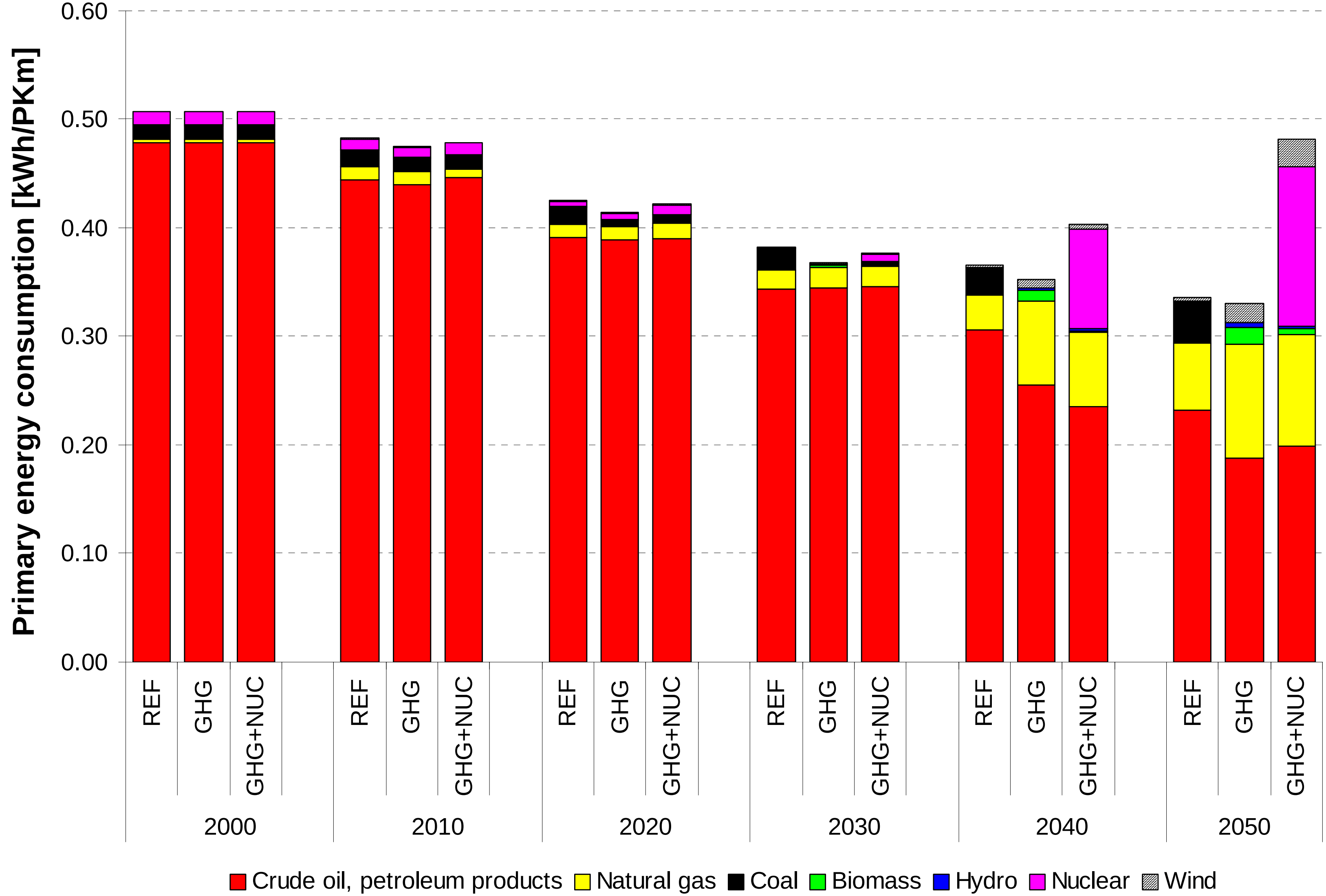
# Balance model: Specific $CO_2$ emissions for electricity



# Balance model: $CO_2$ emissions per Pkm



# Balance model: Primary energy consumption per Pkm



# Duality in linear programming



## Primal Problem

$$\text{Min} \quad c_1x_1 + c_2x_2 + \quad + c_nx_n$$

$$\begin{array}{ccccccccccc}
a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & \geq & b_1 & \perp & y_1 \\
a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & \geq & b_2 & \perp & y_2 \\
\vdots & & \vdots & & \ddots & & \vdots & & \vdots & & \vdots \\
a_{i1}x_1 & + & a_{i2}x_2 & + & \cdots & + & a_{in}x_n & \geq & b_i & \perp & y_i \\
\vdots & & \vdots & & \ddots & & \vdots & & & & \vdots \\
a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & \geq & b_m & \perp & y_m \\
 & & & & & & x_j & \geq & 0 & &
\end{array}$$

## Dual Problem

$$\text{Max} \quad b_1y_1 + b_2y_2 + \quad + b_my_m$$

$$\begin{array}{ccccccccc}
a_{11}y_1 & + & a_{21}y_2 & + & \cdots & + & a_{m1}y_m & \leq & c_1 \\
a_{12}y_1 & + & a_{22}y_2 & + & \cdots & + & a_{m2}y_m & \leq & c_2 \\
\vdots & & \vdots & & \ddots & & \vdots & \leq & \vdots \\
a_{1j}y_1 & + & a_{2j}y_2 & + & \cdots & + & a_{mj}y_m & \leq & c_j \\
\vdots & & \vdots & & \ddots & & \vdots & & \\
a_{1n}y_1 & + & a_{2n}y_2 & + & \cdots & + & a_{mn}y_m & \leq & c_n \\
 & & & & & & y_j & \geq & 0
\end{array}$$

**Decision variables (primal variables):**

- Activities (usage of which technology)
- Energy flows
- Investments

**Equations:**

- Energy/material/emission balances
- Efficiency relationships
- Capacity-activity constraints
- Peaking equation (reserve capacity)
- GHG abatement constraint, Quota for renewables,…

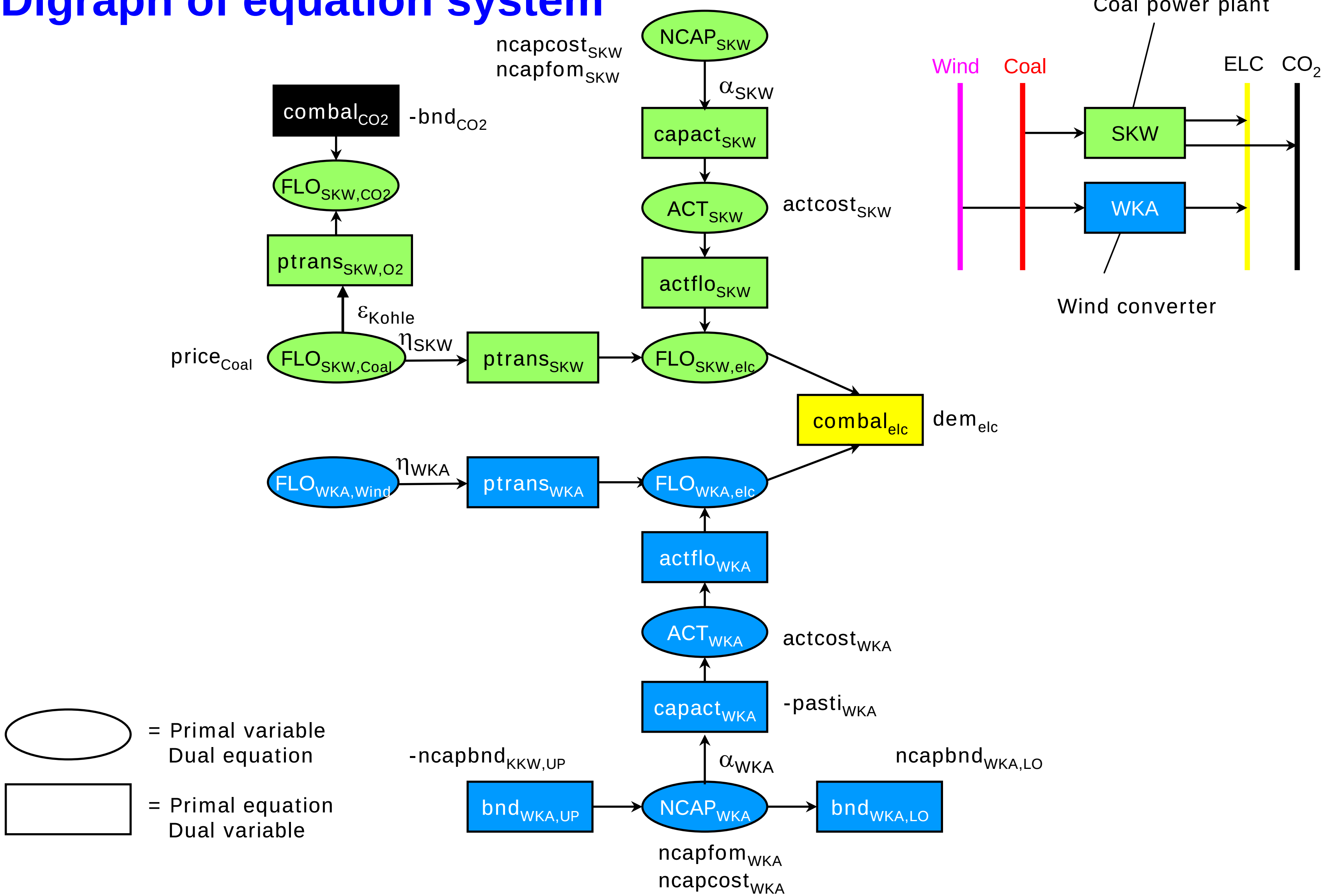
**Decision variables (dual variables):**

- Commodity prices
- Premium for peak consumption, e.g. of electricity
- Opportunity and reduced costs of capacity
- Price for emission permits, renewable certificates

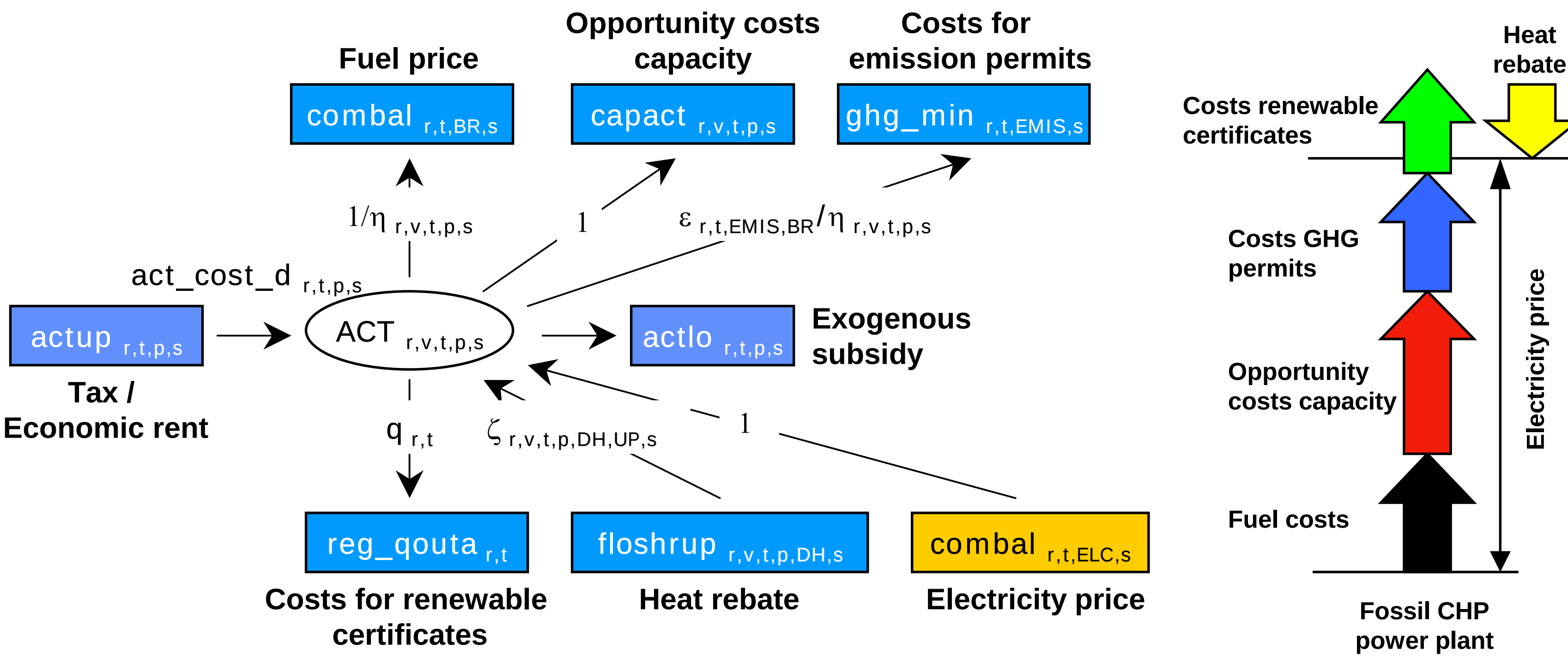
**Equations:**

- Coverage of investment and fixed capacity costs
- Price formation equation for covering the investment costs, variable costs, emission permit costs, etc
- …

# Digraph of equation system



# Dual equation: Covering costs of electricity producing technologies by electricity price

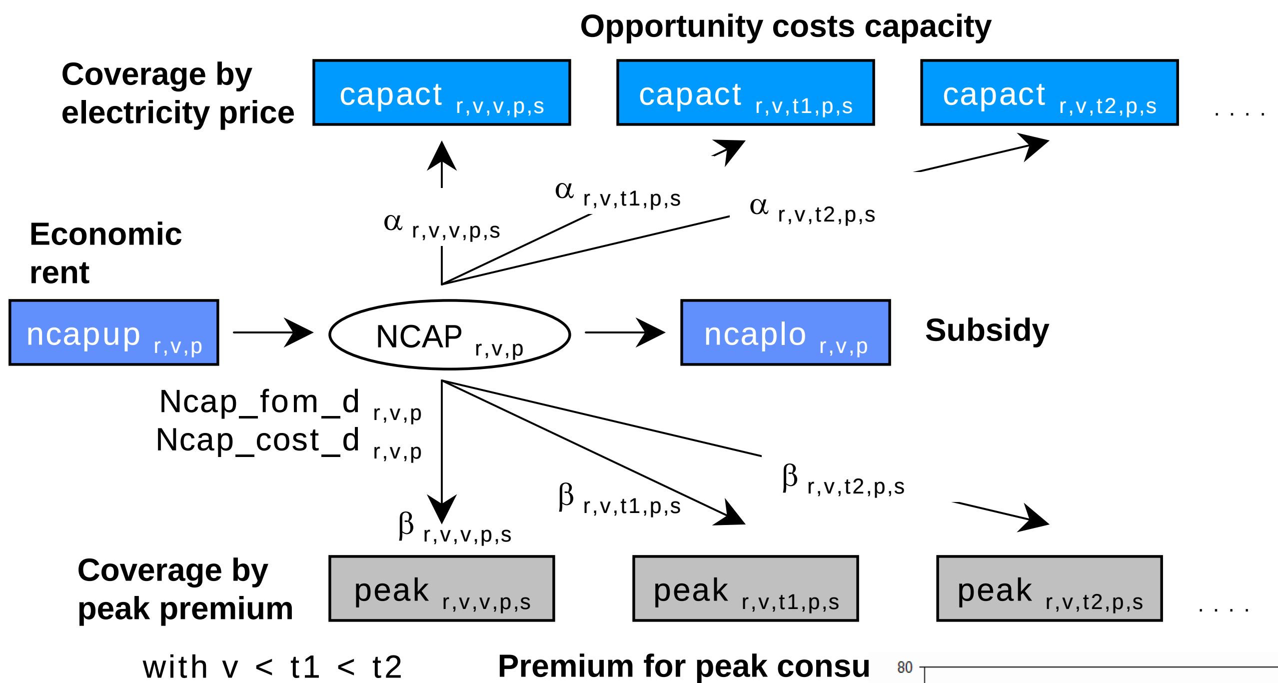


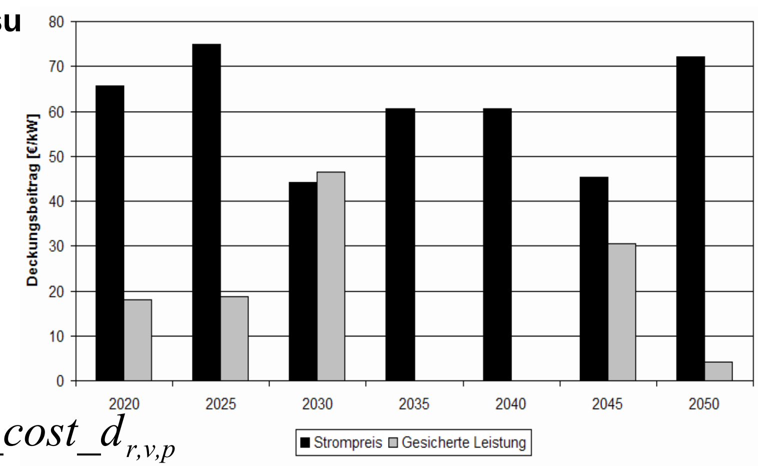

$$ACT_{r,t,p,s}:$$

$$act\_cost\_d_{r,t,p,s} + \frac{1}{\eta_{r,t,p,s}} \cdot combal_{r,t,BR,s} + capact_{r,v,t,p,s} + \frac{\varepsilon_{r,t,emis,BR}}{\eta_{r,v,t,p,s}} ghg\_min_{r,t,EMIS,s} +$$

$$q_{r,t} \cdot reg\_quota_{r,t} - \zeta_{r,v,t,p,DH,UP,s} \cdot floshrup_{r,v,t,p,DH,s} + actlo_{r,t,p,s} - actup_{r,t,p,s} \geq combal_{r,t,ELC,s}$$

# Dual equation: Coverage capacity related costs



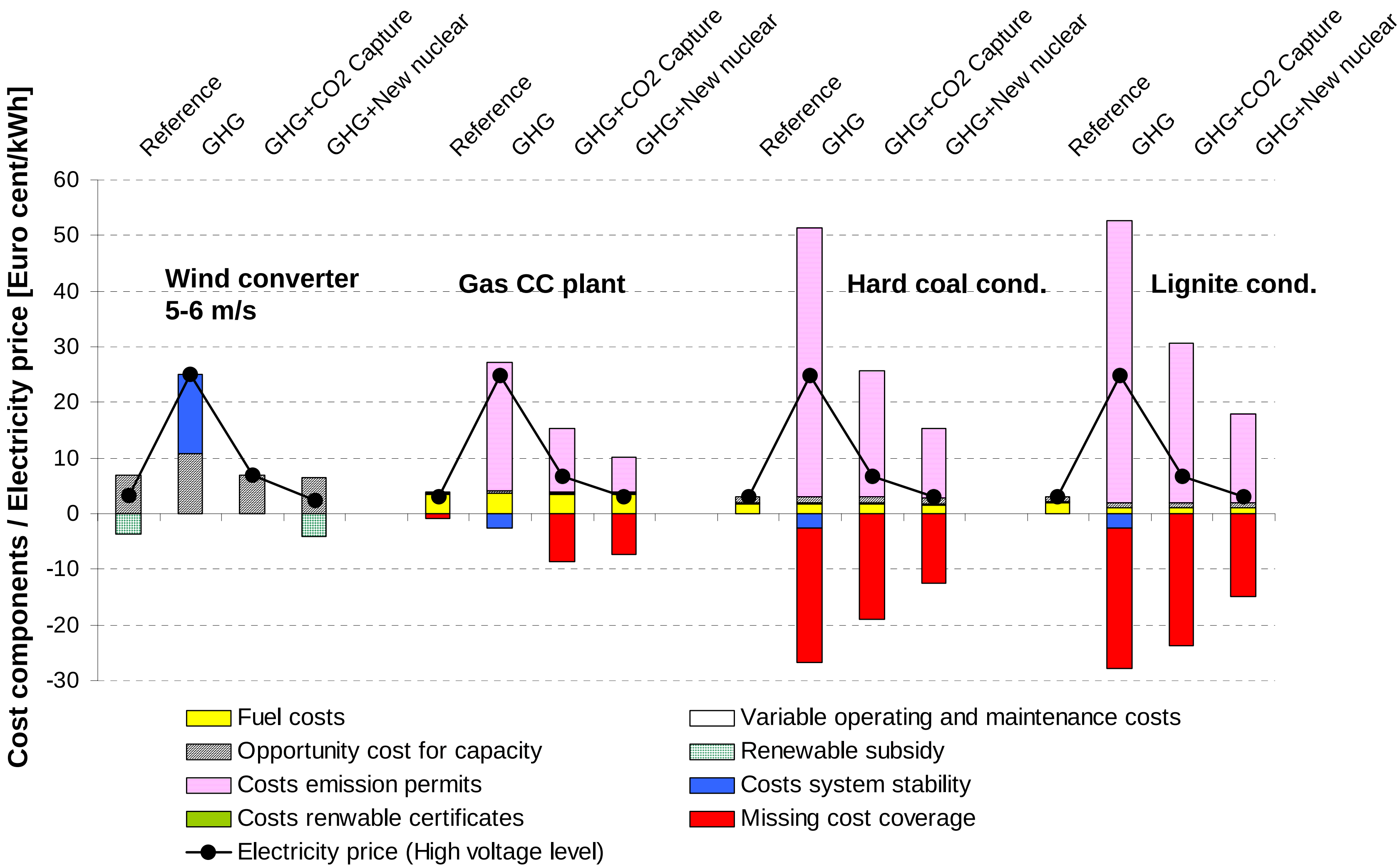

$$NCAP_{r,v,p}:$$

$$\sum_{s\in prcts_{r,p}} \sum_{vintyr_{r,v,t,p}} \alpha_{r,v,t,p,s}\cdot capact_{r,v,t,p,s} +$$

$$\sum_{s\in comts_{r,c}} \sum_{vintyr_{r,v,t,p}} \beta_{r,v,t,p,s}\cdot peak_{r,t,c,s} +$$

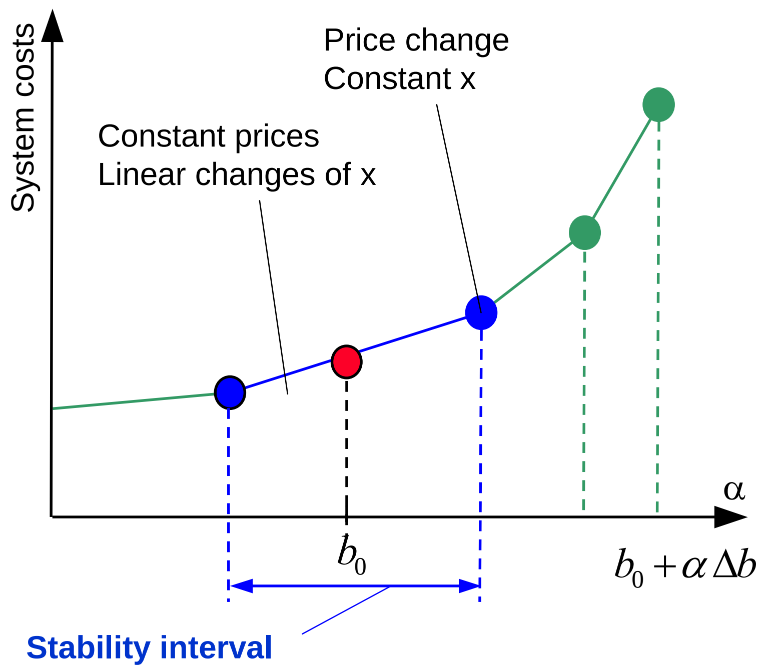
$$ncaplo_{r,v,p} - ncapup_{r,v,p} \le ncap\_fom\_d_{r,v,p} + ncap\_cost\_d_{r,v,p}$$

# Dual equation: Covering technology costs in 2050



# Marginal sensitivity analysis and parametric programming

## Example of variation the RHS b of the primal problem (e. g. GHG target)




**Optimal partition of solution $x^*$ :**

$$Min\{z = c^T x : Ax - w = b, x \geq 0\}$$

$$x_B \in \{x_j^* : x_j > 0\} \qquad w_B \in \{w_j^* : w_j > 0\}$$

$$x_N \in \{x_j^* : x_j = 0\} \qquad w_N \in \{w_j^* : w_j = 0\}$$

**Marginal sensitivity analysis**

Solution of interval problem:

$$\alpha_{\max} = Max\, \alpha$$
$$Ax - w = b_0 + \alpha \Delta b$$
$$x_B \geq 0,\ x_N = 0$$
$$L_B \leq x_B \leq U_B$$
$$w_B \geq 0,\ w_N = 0$$

Application:

- Technology specific abatement costs
- Determining price setting activities (electricity prices, certificate prices)

$$\Rightarrow x_B = f(\alpha) \quad \text{linearfunction}$$

**Parametric programming**

Solution of kink point problem for determining optimal partition in next interval:

$$Min\, c^T x$$
$$Ax - w = \Delta b$$
$$x_N \geq 0,\ x_B \in \Re$$
$$w_N \geq 0,\ w_B \in \Re$$

Application:

- Variation GHG target
- Variation import prices

# Sensitivity analysis: Decomposition of variables

**Sensitivity analysis of RHS parameter $b_i$ :**

$$\Rightarrow \frac{\partial x_{B,j}}{\partial b_i} \quad , j \in \{j : x_j > 0\}$$

$$y_i = \sum_{j \in m} \left( \frac{\partial x_{B,j}}{\partial b_i} \cdot c_j \right)$$ *Shadow price of constraint i*

↑

*Coefficients of jth column of* $\mathbf{B}^{-1}$*:* $\mathbf{y} = \mathbf{c}\mathbf{B}^{-1}$ *with* $\mathbf{A} = \begin{pmatrix} \mathbf{B} & \mathbf{N} \\ \mathbf{B}^* & \mathbf{N}^* \end{pmatrix} \begin{matrix} \Leftrightarrow w_N = 0,\ y_N > 0 \\ \Leftrightarrow w_B > 0,\ y_B = 0 \end{matrix}$

$$\Updownarrow \qquad \Updownarrow$$

$$x_B > 0 \qquad x_N = 0$$

**Sensitivity analysis of cost parameter $c_j$ :**

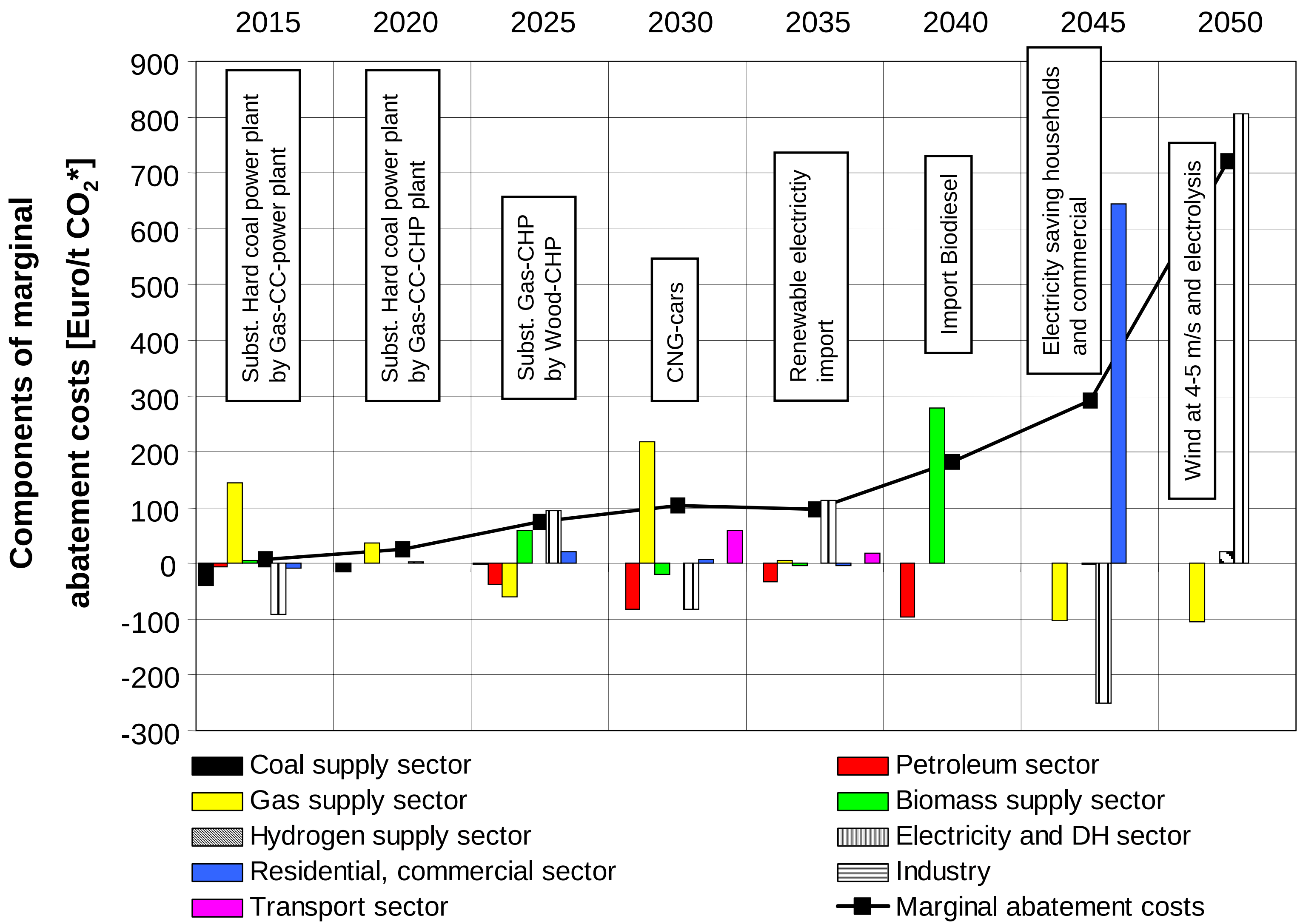
$$\Rightarrow \frac{\partial y_{B,i}}{\partial c_j} \quad , i \in \{i : y_i > 0\}$$

$$x_j = \sum_{i \in n} \left( \frac{\partial y_{B,i}}{\partial c_j} \cdot b_i \right)$$ *Primal variable j*

↑

*Coefficients of ith row of* $\mathbf{B}^{-1}$*:* $x = \mathbf{B}^{-1}\mathbf{b}$

# Sensitivity analysis: Drivers of GHG abatement costs



# Sensitivity analysis: Marginal abatement costs in the reference scenario