

# Optimization of electricity production capacity under uncertainty

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## Abstract

The task of optimal long-term planning of electricity production capacity considering uncertainty intervals of the base, peak and intermediate loads, and multi-stage nature of the planning process will be tackled. Theoretical min-max approach to the problem will be given. Model uses dynamic programming and takes into account also constraints on structure of generating capacity and security of supply.

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## 1. Introduction

Generation expansion planning is an important planning problem for power systems. In the models of integrated energy-economy-environment planning like MARKAL [1] (that is used in Estonia [2], [3]), the long-term planning of electricity generation capacity bases on the requirement to fulfill the electricity consumption forecast. The seasonal and diurnal variations of the power system load are described quite simply and they are derived from the annual consumption. MARKAL uses 3 seasons (winter, intermediate, summer) and differentiates day and night in each season splitting a year so into 6 time divisions. User of the model can determine the lengths of seasons and day/night in the each season. After that the user can define for each energy consumer the distribution of its total annual energy consumption between those 6 time divisions. The load in each time division is calculated by dividing the

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energy consumption in that interval by the length of interval (number of hours in it). As a result, 6 average load levels will represent the annual load curve. To account the peaks of electric load, a special coefficient is used. It shows the amount by which installed capacity exceeds the average load in the time division of maximum demand. Reserve capacity requirements are accounted by determining the coefficients for scheduled and forced outages of power plants. User can define also those power plants that are not able to follow the load (base load plants), but he cannot define the plants that are envisaged for covering only the peak load.

Practice has shown that in some cases this relatively simple description of electric load curve can lead to unrealistic results of generating capacity planning. For example, the model can “build” power plants that will never operate, but serve as reserve only. Balancing of wind power fluctuations by fast peak load power plants (gas turbines etc.) cannot be accounted as well. In addition, the random nature of the power plant characteristics and load are usually neglected in the long-term energy planning models.

The limitations of linear programming (LP) planning tools gave us the motivation to start elaborating the improved optimal power generation planning methodology basing on our years-long experience in the field of optimal load scheduling exercises and methods for accounting the uncertainty of information [4], [5].

The objective of long-term optimization of electricity generation capacity is the minimization of the total costs (expected investment and operational costs) considering the reliability constraints.

This paper will tackle generally the problem of generation planning under conditions of uncertainty by using the nonlinear programming methods. Several publications like [6] and [7] show wide interest to this topic during recent years.

## 2. Optimization of generating power under deterministic conditions

Load demand for active and reactive power is continually changing. The changes of load may be described by load curves or by load duration curves. The load curves are used for the operational and short-term (days, weeks and month) scheduling and the load duration curves for the long-term (years) planning. The examples of the load curve and load duration curve for one year are shown in Fig. 1 and Fig. 2.

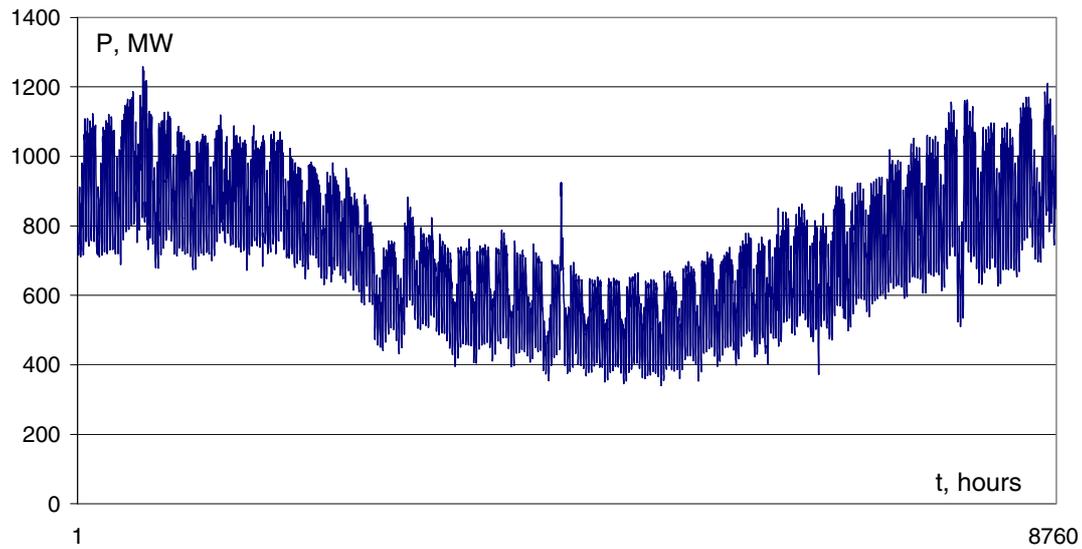


Fig. 1. Annual electric load curve

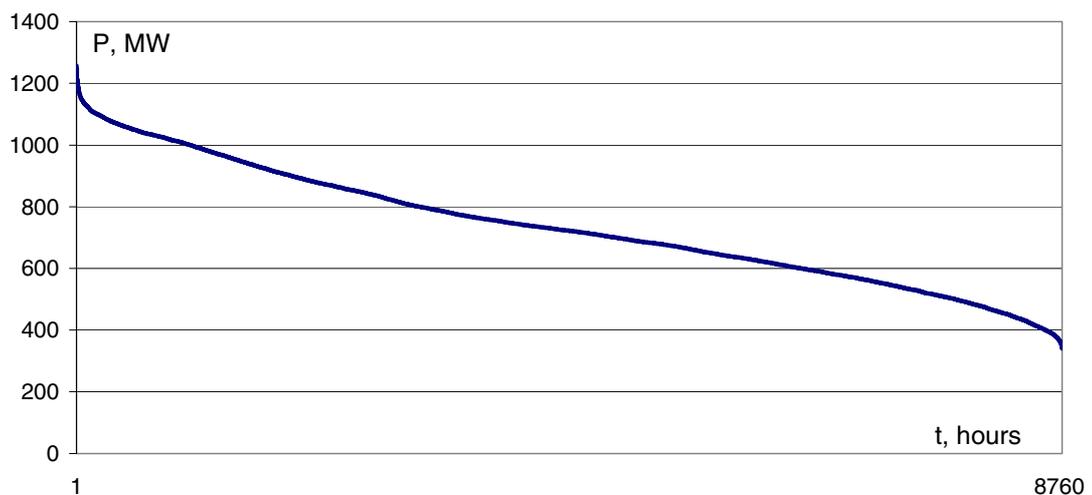


Fig. 2. Annual load duration curve

The power system load is divided usually into 3 categories:

- 1) Base-load (duration time 8760 hours)
- 2) Intermediate load (duration time from 2000 to 8760 hours)
- 3) Peak-load (duration time up to 2000 hours).

In the Estonian power system, the base-load forms about 35%, intermediate load about 40% and peak-load about 25% from the maximum load.

The power system must have sufficient active and reactive power generating capacity to cover the load changes since the electricity cannot be conveniently stored in sufficient quantities. Therefore the power system must have the following types of generating units:

- 1) Base-load generating units
- 2) Intermediate load generating units
- 3) Peak-load generating units
- 4) Frequency and power control units.

The structure of generating units has to be optimal. It is optimal if the requirements of power balance, reserve capacity, reliability, security, and other constraints are fulfilled with minimum total costs.

Let us suppose that the set of existing power plant units and the set of units possible to build in a future are given. Then the optimization task is to determine the optimal unit commitment and the economic scheduling of units' active power and energy production during the planning period.

The objective of the optimization task is to minimize the sum of operational costs and expected investment costs for the each year of planning period considering the reliability and environmental constraints.

We assume that the following initial information is given:

1. Annual load demand duration curve  $P_D(t)$ , where  $t$  is the duration of load. At that the load consists of the net load and transmission and distribution losses.
2. Functions of total costs for each generating unit:

$$C_i(P_i) = C_{if} + C_{iv}(P_i), \quad (1)$$

where  $C_{if}$  - fixed costs,  $C_{iv}(P_i)$  - variable costs.

The functions (1) must be given for all existing and possible new generation units.

Let us examine now the long-term multiyear generating capacity planning basing on the active power optimization problem for one year. To describe the principles with less complex formulas, we omit here the power plants with limited primary energy resources, e.g. hydro plants with seasonal reservoirs. They would bring some additional constraints and iterations described in [4].

The optimization problem can be written in the following form:

$$\text{Minimize} \quad \int_{t=0}^{T^{\max}} \sum_{i=1}^N C_i(P_i(t)) dt \quad (2)$$

$$\text{Subject to:} \quad P_D(t) - \sum_{i=1}^N P_i(t) = 0, \quad t = [0, T^{\max}] \quad (3)$$

$$P_i^{\min} \leq P_i(t) \leq P_i^{\max}, \quad i = 1, \dots, N, \quad t = [0, T^{\max}] \quad (4)$$

$$\sum_{i=1}^N P_i^{\max}(t) - P_D(t) \geq P_{reserve}(t), \quad t = [0, T^{\max}]. \quad (5)$$

Here

$P_i(t)$  - load duration curve of  $i^{\text{th}}$  generating unit,

$N$  - total number of generating units possible to use for optimization,

$P_{reserve}(t)$  - duration curve of needed reserve capacity,

$T^{max}$  – length of planning period.

The problem (2)-(5) includes the unit commitment and economic dispatch problems. The inequalities (4) consider also capacity changes and lifetimes of units.

The objective is to determine the following functions:

- 1) optimal load duration curves of units  $P_i(t), i=1, \dots, N$ ,
- 2) optimal cost duration curves of units  $C_i(P_i(t)), i=1, \dots, N$ ,
- 3) reserve capacity duration curves of units  $P_{irc}(t), i=1, \dots, N$ ,
- 4) annual total energy production for every unit  $W_i, i=1, \dots, N$ ,
- 5) annual total costs for every unit  $C_{i\Sigma}, i=1, \dots, N$ .

The solution algorithm of the problem is following (further called algorithm A1):

1. Determination of the optimal unit commitment when  $P_D(t) = P_D^{min}$  (base load situation) with taking into account reserve requirement (5).
2. Economic load dispatch when  $P_D(t) = P_D^{min}$  using the  $\mu$ -iteration method.

The necessary optimality conditions are:

$$\frac{\partial C_i}{\partial P_i} = \mu \text{ for } P_i^{min} < P_i < P_i^{max}, \quad (6)$$

$$\frac{\partial C_i}{\partial P_i} \leq \mu \text{ for } P_i = P_i^{max}, \quad (7)$$

$$\frac{\partial C_i}{\partial P_i} \geq \mu \text{ for } P_i = P_i^{min}, \quad (8)$$

where

$\frac{\partial C_i}{\partial P_i}$  - incremental cost of unit,

$\mu$  - LaGrange multiplier (i.e. incremental cost for power system).

3. Calculation of the composite cost function of units up to  $P_D(t) = P_D^{\min}$ .

The composite cost curve for set of units is the function of total costs calculated as following [8]:

$$C_{\Sigma N}(P_{\Sigma}) = C_1(P_1) + \dots + C_N(P_N), \quad (9)$$

where

$$P_{\Sigma} = P_1 + \dots + P_N, \quad (10)$$

$$\frac{\partial C_i}{\partial P_i} = \frac{\partial C_2}{\partial P_2} = \dots = \frac{\partial C_N}{\partial P_N} = \mu. \quad (11)$$

4. Determination of unit commitment when  $P_D(t) = P_D^{\min} + \Delta P_D$ .
5. Economic load dispatch when  $P_D(t) = P_D^{\min} + \Delta P_D$ .
6. Calculation of the composite cost function of generating units up to the demand value  $P_D(t) = P_D^{\min} + \Delta P_D$ .
7. Step-wise increase of demand load value and the stages 4, 5 and 6 will be repeated with determination of the need for a next generating unit until  $P_D(t) = P_D^{\max}$  (highest peak load situation).

The critical values of demand load, when it is economical to add the next generating unit, can be calculated from equation:

$$C_{\Sigma n}(P_{\Sigma}) = C_{\Sigma n+1}(P_{\Sigma}) \quad (12)$$

where n – number of units.

### 3. Optimization of generating power under uncertain conditions

#### Uncertainty factors

The main uncertainty factors in the model (2)–(5) are:

- 1) load demand duration curve  $P_D(t)$ ,

2) functions of total costs for every generating unit  $C_i(P_i)$ .

These uncontrollable factors are considered below in detail.

Two kinds of uncertainty can exist [4], [5]:

- 1) Deterministic-uncertain information – uncertainty zones of factual values of functions or parameters are known.
- 2) Probabilistic-uncertain information – probabilistic characteristics of object are not known exactly, but in the form of uncertainty zones.

In this paper we will consider only deterministic-uncertain information.

Let the load duration curve  $P_D(t)$  be given in the form of intervals (Fig. 3):

$$P_D^{\min}(t) \leq P_D(t) \leq P_D^{\max}(t), \quad 0 \leq t \leq T \quad (13)$$

where the functions  $P_D^{\min}(t)$  and  $P_D^{\max}(t)$  are given.

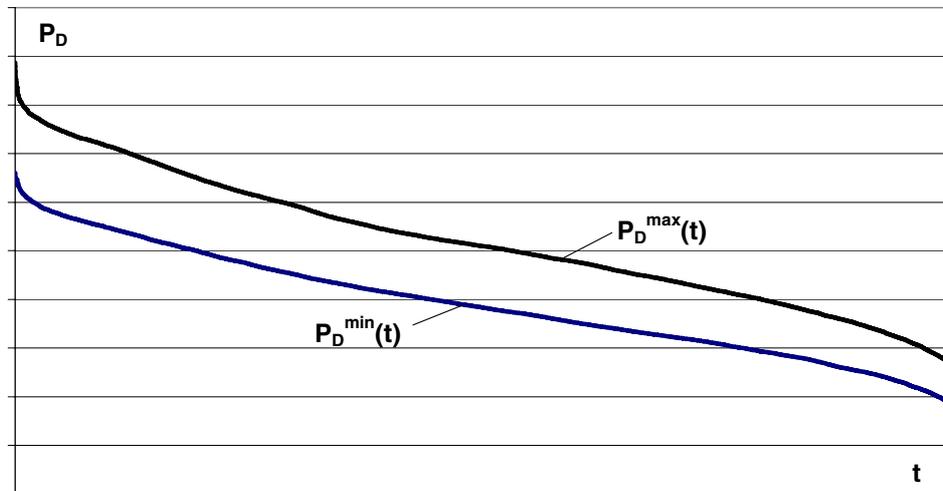


Fig. 3. Load demand duration curve in the uncertainty form

The cost functions of generating units are given in the form of intervals too:

$$C_i^{\min}(P_i) \leq C_i(P_i) \leq C_i^{\max}(P_i), \quad (14)$$

where the functions  $C_i^{\min}(P_i)$  and  $C_i^{\max}(P_i)$  are given (Fig. 4).

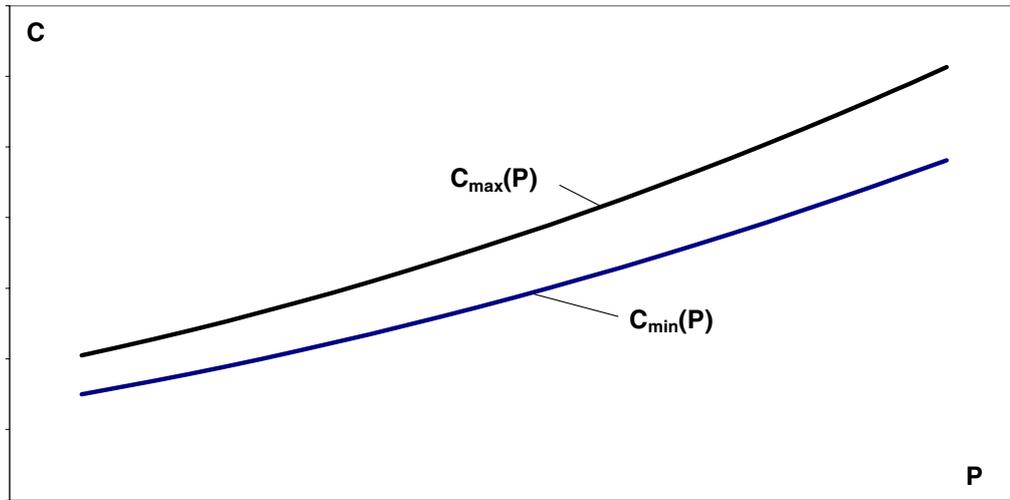


Fig. 4. Unit cost function in the uncertainty form

#### Min-max model

The most well founded criterion for optimal scheduling of active loads in power system under uncertainty is the criterion of min-max risk or possible losses caused by uncertainty of information [4], [5]:

$$\min_{\bar{P}(t)} \max_{\tilde{Z}(t)} \int_0^T R(\bar{P}(t), \tilde{Z}(t)) dt, \quad (15)$$

where

$R$  – function of risk or possible losses caused by uncertainty factors:

$$R(\bar{P}(t), \tilde{Z}(t)) = C_{\Sigma}(\bar{P}(t), \tilde{Z}(t)) - \min C_{\Sigma}(P(t), \tilde{Z}(t)), \quad (16)$$

$\bar{P}(t)$  – vector of planned load duration curves of units,

$\tilde{Z}(t)$  – vector of uncertain factors,

$C_{\Sigma}$  – actual total costs of power units,

$\min C_{\Sigma}$  – minimum of total costs if we could have the exact deterministic information about uncertainty factors.

Operator  $\min \max R$  means the minimization of maximum risk caused by uncertainty factors.

### Optimality conditions

The optimality conditions of min-max problem arise from the main theorem of the game theory and can be expressed as follows:

if the  $(\bar{P}^0(t))$  is the optimal plan for  $\min \max R$  criterion, then

$$R(\bar{P}^0(t), Z^-(t)) = R(\bar{P}^0(t), Z^+(t)). \quad (17)$$

In general case, it is necessary to solve the problem

$$\min_{\bar{P}(t)} \max_{\Omega} \int_0^T E R(\bar{P}(t), \tilde{Z}(t)) dt, \quad (18)$$

where  $E$  is expected value of risk,  $\Omega$  – a set of mixed strategy of uncertain factors.

It is possible to compose the deterministic equivalent of min-max problem on the base of given above conditions. It requires the finding of the min-max load demand curves and cost functions of generating units. If we replace the deterministic curves by the min-max curves, we can use the initial deterministic model for calculating the min-max optimal results.

The algorithm of solution of the min-max problem is similar to the algorithm A1.

The main differences are:

- 1) determination of min-max optimal unit commitment,
- 2) determination of min-max optimal load scheduling,
- 3) calculation of the deterministic equivalents of composite cost functions of generating units,
- 4) calculation of minimum and maximum load duration curves and cost duration curves of units.

The min-max risk optimality of unit commitment means the unit combinations for each hour that guarantee fulfilling of the requirement of equation (17). The optimal unit commitment problem has many constraints. The most suitable solution methods of it are the priority-list schemes and dynamic programming. The min-max optimization of unit commitment can reduce the maximum risk caused by uncertainty of information about 2-3 times.

#### **4. Conclusions**

Long-term planning of electricity generation is very important problem. This paper describes some main principles of the research in this field carried out at the Tallinn University of Technology.

The deterministic planning models enable to analyze single cases, but it can be insufficient for making optimal development decisions because the initial information about the loads, power plant characteristics and costs, etc. has random nature.

Min-max optimization models enable to take into account the uncertainty of uncontrollable factors and to minimize the maximum possible economic loss caused by uncertainty. This makes decision-making more justified.

Using of min-max optimization enables to reduce possible economic loss caused by uncertainty of information several times.

Generating capacity planning could use also load duration curves determined for shorter periods than whole year, like separate curves for winter, spring-autumn and summer. It could possibly improve the quality of optimization results.

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