

# Optimization of electricity production capacity under uncertainty

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## Reasons of TUT interest to methodology of long-term generation capacity planning

- The limitations of linear programming (LP) planning tools.
- Need for consideration of non-linear characteristics of power system objects.
- Need for accounting the random nature and incompleteness of initial information about loads and characteristics of objects.
- Years-long experience in the field of optimal electric load scheduling exercises and methods for accounting the uncertainty of information.

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## Objective

- The objective of long-term optimization of electricity generation capacity is the minimization of the total costs (expected investment and operational costs) considering the reliability and environmental constraints.
- This presentation will tackle generally the problem of generation planning under conditions of uncertainty by using the nonlinear programming (NLP) methods.

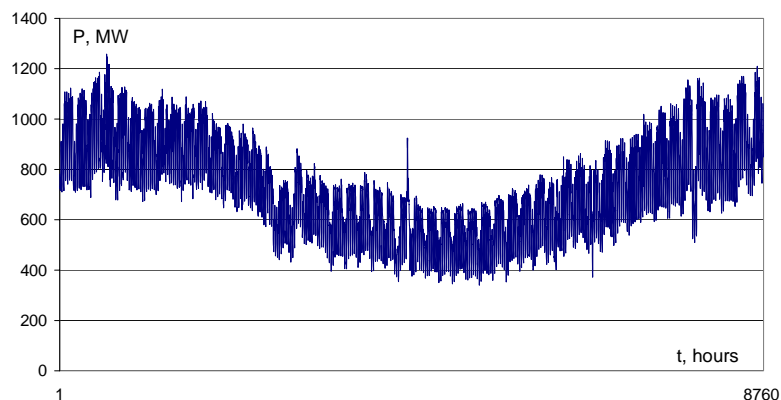
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## The power system load is described....

by load curves used for the operational and short-term (days, weeks and month) scheduling



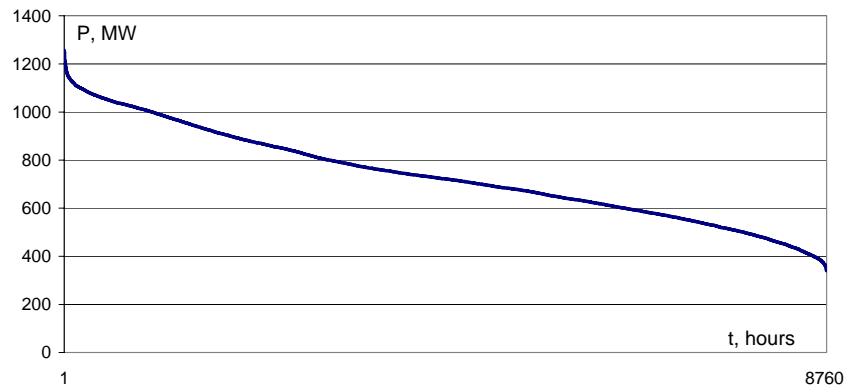
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## The power system load is described....

by load duration curves used for the long-term (years) planning



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## The power system load is divided usually into 3 categories:

- Base-load (duration time 8760 hours)
- Intermediate load (duration time from 2000 to 8760 hours)
- Peak-load (duration time up to 2000 hours)

In the Estonian power system, the base-load forms about 35%, intermediate load about 40% and peak-load about 25% from the maximum load.

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## The power system must have the following types of generating units:

- Base-load generating units
- Intermediate load generating units
- Peak-load generating units
- Frequency and power control units

The structure of generating units is optimal if the requirements of power balance, reserve capacity, reliability, security, and other constraints are fulfilled with minimum total costs.

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## Optimization of generating power under deterministic conditions (1)

- The optimization task is to determine the optimal unit commitment and the economic scheduling of units' active power and energy production during the planning period.
- The objective of the optimization task is to minimize the sum of operational costs and expected investment costs for the each year of planning period considering the reliability and environmental constraints.
- We assume the set of existing power plant units and the set of units possible to build in a future are given.

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## Optimization of generating power under deterministic conditions (2)

We assume that the following initial information is given:

- Annual load demand duration curve  $P_D(t)$ , where  $t$  is the duration of load .
- Functions of total costs for each existing and possible new generating unit:

$$C_i(P_i) = C_{if} + C_{iv}(P_i)$$

where  $C_{if}$  – fixed costs and  $C_{iv}(P_i)$  – variable costs

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## Optimization of generating power under deterministic conditions (3)

The optimization problem is:

- Minimize 
$$\int_{t=0}^{T^{\max}} \sum_{i=1}^N C_i(P_i(t)) dt$$

- Subject to:

$$P_D(t) - \sum_{i=1}^N P_i(t) = 0 \quad t=[0, T^{\max}]$$

$$P_i^{\min} \leq P_i(t) \leq P_i^{\max} \quad i=1, \dots, N, \quad t=[0, T^{\max}]$$

$$\sum_{i=1}^N P_i^{\max}(t) - P_D(t) \geq P_{reserve}(t) \quad t=[0, T^{\max}]$$

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## Optimization of generating power under deterministic conditions (4)

The objective is to determine the following functions:

- optimal load duration curves of units  $P_i(t), i=1, \dots, N$
- optimal cost duration curves of units  $C_i(P_i(t)), i=1, \dots, N$
- reserve capacity duration curves of units  $P_{irc}(t), i=1, \dots, N$
- annual total energy production for every unit  $W_p, i=1, \dots, N$
- annual total costs for every unit  $C_{i\Sigma}, i=1, \dots, N$

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## Optimization of generating power under deterministic conditions (5)

The solution algorithm (algorithm A1):

1. Determination of the optimal unit commitment when  $P_D(t) = P_D^{min}$  (base load situation) with taking into account reserve requirement
2. Economic load dispatch when  $P_D(t) = P_D^{min}$  using the  $\mu$ -iteration method
3. Calculation of the composite cost function of units up to  $P_D(t) = P_D^{min}$  and the composite cost curve for set of units

$$C_{\Sigma N}(P_{\Sigma}) = C_1(P_1) + \dots + C_N(P_N)$$

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## Optimization of generating power under deterministic conditions (6)

### The solution algorithm (continued):

4. Determination of the optimal unit commitment when  $P_D(t) = P_D^{min} + \Delta P_D$
5. Economic load dispatch when  $P_D(t) = P_D^{min} + \Delta P_D$
6. Calculation of the composite cost function of generating units up to the demand value  $P_D(t) = P_D^{min} + \Delta P_D$
7. **Step-wise increase of demand load value and the stages 4, 5 and 6 will be repeated** with determination of the need for a next generating unit until  $P_D(t) = P_D^{max}$  (highest peak load situation).

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## Optimization of generating power under deterministic conditions (7)

### The solution algorithm (continued):

The critical values of demand load, when it is economical to add the next generating unit, can be calculated from equation:

$$C_{\Sigma n}(P_{\Sigma}) = C_{\Sigma n+1}(P_{\Sigma})$$

where  $n$  – number of units

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## Optimization of generating power under uncertain conditions (1)

### Uncertainty factors:

- load demand duration curve  $P_D(t)$
- functions of total costs for every generating unit  $C_i(P_i)$

Here we will consider only deterministic-uncertain information, i.e. uncertainty zones of factual values of functions or parameters are known

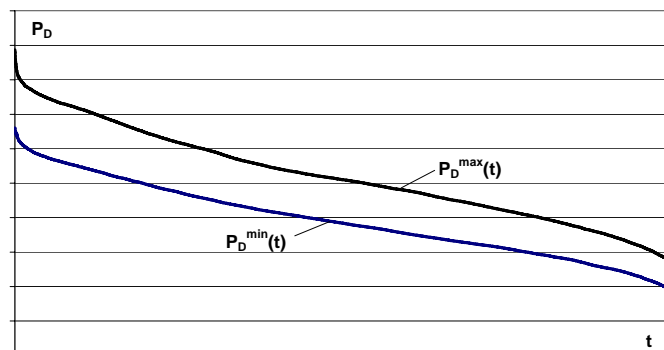
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## Optimization of generating power under uncertain conditions (2)

Uncertain load duration curve:  $P_D^{\min}(t) \leq P_D(t) \leq P_D^{\max}(t)$



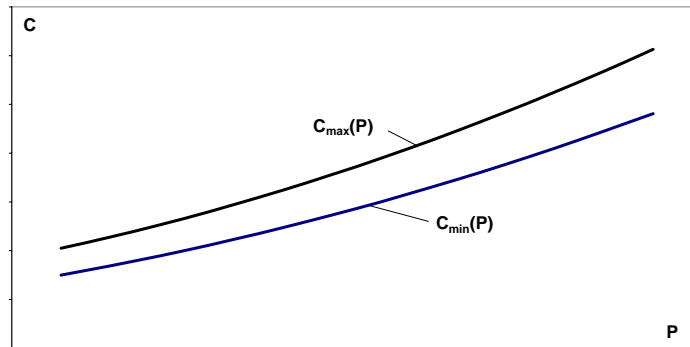
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## Optimization of generating power under uncertain conditions (3)

Uncertain cost function:  $C_i^{\min}(P_i) \leq C_i(P_i) \leq C_i^{\max}(P_i)$



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## Optimization of generating power under uncertain conditions (4)

Min-max model:

The most well founded criterion is the minimization of maximum risk caused by uncertainty of information

$$\min_{\bar{P}(t)} \max_{\tilde{Z}(t)} \int_0^T R(\bar{P}(t), \tilde{Z}(t)) dt$$

where  $R$  – function of risk or possible losses caused by uncertainty factors:

$$R(\bar{P}(t), \tilde{Z}(t)) = C_{\Sigma}(\bar{P}(t), \tilde{Z}(t)) - \min C_{\Sigma}(P(t), \tilde{Z}(t))$$

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## Optimization of generating power under uncertain conditions (5)

Min-max model (continued):

$\bar{P}(t)$  – vector of planned load duration curves of units,

$\tilde{Z}(t)$  – vector of uncertain factors,

$C_{\Sigma}$  – actual total costs of power units,

$\min C_{\Sigma}$  – minimum of total costs if we could have the exact deterministic information about uncertainty factors.

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## Optimization of generating power under uncertain conditions (6)

Optimality conditions:

If the  $(\bar{P}^0(t))$  is the optimal plan for **min max R** criterion, then

$$R(\bar{P}^0(t), Z^-(t)) = R(\bar{P}^0(t), Z^+(t))$$

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## Optimization of generating power under uncertain conditions (7)

### Optimality conditions (continued):

In general case, it is necessary to solve the problem

$$\min_{\bar{P}(t)} \max_{\Omega} \int_0^T E R(\bar{P}(t), \tilde{Z}(t)) dt$$

where

$E$  – expected value of risk,

$\Omega$  – a set of mixed strategy of uncertain factors.

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## Optimization of generating power under uncertain conditions (8)

### Deterministic equivalent:

It is possible to compose the deterministic equivalent of min-max problem on the base of given above conditions.

It requires the finding of the min-max load demand curves and cost functions of generating units. If we replace the deterministic curves by the min-max curves, we can use the initial deterministic model for calculating the min-max optimal results.

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## Optimization of generating power under uncertain conditions (9)

The algorithm of solution of the min-max problem is similar to the algorithm A1 :

The main differences are:

1. determination of min-max optimal unit commitment using the priority-list schemes and dynamic programming,
2. determination of min-max optimal load scheduling,
3. calculation of the deterministic equivalents of composite cost functions of generating units,
4. calculation of minimum and maximum load duration curves and cost duration curves of units.

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## Conclusions

- The deterministic planning models enable to analyze single cases, but it can be insufficient for making optimal development decisions because the initial information about the loads, power plant characteristics and costs, etc. has random nature.
- Min-max optimization models enable to take into account the uncertainty of uncontrollable factors and to minimize the maximum possible economic loss caused by uncertainty. This makes decision-making more justified.
- Using of min-max optimization enables to reduce possible economic loss caused by uncertainty of information several times.

O.Liik, R.Oidram, M. Keel. IEW presentation. 24-26 June 2003, IIASA

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